Online Appendix of
Twin Peaks: Covid-19 and the Labor Market

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1 Solving the model under lockdown

In the notation that follows, variables with a superscript L denote variables pertaining to the model under lockdown. Those without such superscripts refer to variables coming from the baseline model without lockdown described in Section 2 of Bradley et al. (2020).

1.1 Surplus functions of the lockdown model

This section proceeds by listing and describing the value functions associated with workers and vacancies of particular types. We then derive the corresponding surplus equations.

Retired workers

A retired worker that has recovered has value that comes from four sources. The first is their flow benefit of retirement. The second is the value associated with death by natural causes. The third is their value associated with the lockdown period ending, in which case they receive the baseline value of their status. The fourth is their continuation value. The formal representation follows

$$rR_{rt}^L = b_o + \chi(0 - R_{rt}^L) + \Lambda(R_r - R_{rt}^L) + \dot{R}_{rt}^L$$

An infected retired worker faces five terms associated with their value function. In addition to their flow benefit and continuation value, they face the value changes associated with the possibilities of death by natural causes, death by COVID-19, recovery and the economy leaving lockdown. The expression is given by

$$rR_{it}^L = b_o + (\chi + \gamma_o(\ell_{it}))(0 - R_{it}^L) + \rho_o(R_{rt}^L - R_{it}^L) + \Lambda(R_{it} - R_{it}^L) + \dot{R}_{it}^L$$

A retired worker that is susceptible receives a flow value, continuation value, faces a possibility of death and infection and the changes from the economy leaving lockdown. The value function can be written as

$$rR_{st}^L = b_o + \chi(0 - R_{st}^L) + \lambda_0^L \ell_{it}(R_{it}^L - R_{st}^L) + \Lambda(R_{st} - R_{st}^L) + \dot{R}_{st}^L.$$

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Recovered young workers

The value of unemployment for a young worker is comprised of six terms. The first, the flow benefit of unemployment, is the same as in the baseline model. Their option value of finding work is now split into two terms, capturing the two possibilities of matching with a firm in the locked and unlocked sectors. The worker can retire at the rate η and the economy can come out of lockdown, in which case the worker receives the value of being recovered in the baseline model. They also receive their continuation value. The formal representation is given below

$$\begin{aligned} rU_{rt}^L = b_u &+ \phi_t \beta \pi \int\int \max\{S_{rt}^L(\alpha, x; L), 0\} d^2 F(\alpha, x) \\ &+ \phi_t \beta (1-\pi) \int\int \max\{S_{rt}^L(\alpha, x; U), 0\} d^2 F(\alpha, x) \\ &+ \eta(R_{rt}^L - U_{rt}^L) + \Lambda(U_{rt} - U_{rt}^L) + \dot{U}_{rt}^L \end{aligned}$$

A recovered worker who is employed now has two separate values, depending on whether their job is in the locked or unlocked sector. A worker with match characteristics (α, x) has contract with wage w and stipulation $m \in \{0, 1\}$ where 1 means working away from home and 0 means working from home. If the match is in the locked sector, then the value for being employed is given by

$$\begin{aligned} rW_{rt}^L(w, \alpha, x, m; L) = w &+ \delta(U_{rt}^L - W_{rt}^L(w, \alpha, x, m; L)) + \eta(R_{rt}^L - W_{rt}^L(w, \alpha, x, m; L)) \\ &+ \nu \left(\beta \max\{S_{rt}^L(\alpha, x, 0; L), S_{rt}^L(\alpha, x, 1; L), 0\} + U_{rt}^L - W_{rt}^L(w, \alpha, x; L)\right) \\ &+ \Lambda(W_{rt}(w, \alpha, x, m) - W_{rt}^L(w, \alpha, x, m; L)) + \dot{W}_{rt}^L(w, \alpha, x, m; L) \end{aligned}$$

where the argument L inside the parentheses signifies that the job is in the locked sector. Notice that, although the working arrangement regarding m can be negotiated in the contract, a match that is locked is unable to operate away from home for the duration of the lockdown. However, once the lockdown ends, we assume that they immediately start producing using the contracted arrangement regarding m. A match that is unlocked has the value

$$\begin{aligned} rW_{rt}^L(w, \alpha, x, m; U) = w &+ \delta(U_{rt}^L - W_{rt}^L(w, \alpha, x, m; U)) + \eta(R_{rt}^L - W_{rt}^L(w, \alpha, x, m; U)) \\ &+ \nu \left(\beta \max\{S_{rt}^L(\alpha, x, 0; U), S_{rt}^L(\alpha, x, 1; L), 0\} + U_{rt}^L - W_{rt}^L(w, \alpha, x; U)\right) \\ &+ \Lambda(W_{rt}(w, \alpha, x, m) - W_{rt}^L(w, \alpha, x, m; U)) + \dot{W}_{rt}^L(w, \alpha, x, m; U), \end{aligned}$$

where this match will involve working away from home during the lockdown when the contract has $m = 1$.

Value of a filled vacancy with a recovered worker

The value of a filled job is affected directly by whether or not the job is locked. A locked job has flow value that comes from home production less the wages paid to the employee. The match will break when either the exogenous separation shock is realised, or if the worker retires. It has option value associated with re-negotiation as well as with the economy leaving lockdown; there is also an associated continuation value. The formal expression is given by

$$\begin{aligned} rJ_{rt}^{L}(w,\alpha,x,m;L) &= p(\alpha,x,0) - w + (\delta+\eta)(V_t^L - J_{rt}^L(w,\alpha,x,m;L)) \\ &+ \nu\left((1-\beta)\max\{S_{rt}^L(\alpha,x,0;L), S_{rt}^L(\alpha,x,1;L),0\} + V_t^L - J_{rt}^L(w,\alpha,x,m;L)\right) \\ &+ \Lambda(J_{rt}(w,\alpha,x,m) - J_{rt}^L(w,\alpha,x,m;L)) + \dot{J}_{rt}^L(w,\alpha,x,m;L). \end{aligned}$$

where notice that V_t^L denotes the value of a vacancy under lockdown. A match that's in the unlocked sector may either be producing from home or away from home depending on the characteristics of the match; it faces no restriction. The value function for an unlocked match for contract stipulation m is

$$\begin{aligned} rJ_{rt}^{L}(w,\alpha,x,m;U) &= p(\alpha,x,m) - w + (\delta+\eta)(V_t^L - J_{rt}^L(w,\alpha,x,m;U)) \\ &+ \nu\left((1-\beta)\max\{S_{rt}^L(\alpha,x,0;U), S_{rt}^L(\alpha,x,1;U),0\} + V_t^L - J_{rt}^L(w,\alpha,x,m;U)\right) \\ &+ \Lambda(J_{rt}(w,\alpha,x,m) - J_{rt}^L(w,\alpha,x,m;U)) + \dot{J}_{rt}^L(w,\alpha,x,m;U) \end{aligned}$$

where notice that the match's output varies with m.

Surplus of a match with a recovered worker

Imposing the equilibrium free entry condition, that $V_t^L = 0$, then combining expressions for retired workers, young workers and filled vacancy value functions gives the surplus equation

$$\begin{aligned} (r+\delta+\eta+\nu+\Lambda)S_{rt}^L(\alpha,x,m;L) &= p(\alpha,x,0) - b_u \\ &- \phi_t\beta\pi\int\int \max\{S_{rt}^L(\alpha,x,0;L), S_{rt}^L(\alpha,x,1;L),0\}d^2F(\alpha,x) \\ &- \phi_t\beta(1-\pi)\int\int \max\{S_{rt}^L(\alpha,x,0;U), S_{rt}^L(\alpha,x,1;U),0\}d^2F(\alpha,x) \\ &+ \nu\max\{S_{rt}^L(\alpha,x,0;L), S_{rt}^L(\alpha,x,1;L),0\} \\ &+ \Lambda S_{rt}(\alpha,x,m) + \dot{S}_{rt}^L(\alpha,x,m;L) \end{aligned}$$

where again notice that the output comes from home production irrespective of the contractual choice of $m \in \{0,1\}$. Then the surplus for an unlocked match with m is given by

$$
\begin{aligned}
(r+\delta+\eta+\nu+\Lambda)S^L_{rt}(\alpha,x,m;U) &= p(\alpha,x,m)-b_u \\
&- \phi_t\beta\pi\int\int \max\{S^L_{rt}(\alpha,x,0;L),S^L_{rt}(\alpha,x,1;L),0\}d^2F(\alpha,x) \\
&- \phi_t\beta(1-\pi)\int\int \max\{S^L_{rt}(\alpha,x,0;U),S^L_{rt}(\alpha,x,1;U),0\}d^2F(\alpha,x) \\
&+ \nu\max\{S^L_{rt}(\alpha,x,0;U),S^L_{rt}(\alpha,x,1;U),0\} \\
&+ \Lambda S_{rt}(\alpha,x,m)+\dot{S}^L_{rt}(\alpha,x,m;U).
\end{aligned}
$$

Infected young workers

An unemployed infected worker's value function contains six terms. They receive their flow value of unemployment and continuation value. They receive value associated with the possibility of moving to recovered status, from dying from the virus, from retiring and from the economy leaving the state of lockdown. The formal expression is given by

$$
rU^L_{it} = b_u + \rho_y\left(U^L_{rt}-U^L_{it}\right) + \gamma_y(\ell_{it})(0-U^L_{it}) + \eta(R^L_{it}-U^L_{it}) + \Lambda(U_{it}-U^L_{it}) + \dot{U}^L_{it}.
$$

An employed worker with infection status is at home on sick pay. They produce no output but continue to receive their contracted-upon wage; upon recovery, they return back to production for the firm. Consequently, whether the infected worker's match is in the locked or unlocked sector affects their value function. A worker in the locked sector receives value from their wage, continuation value, value associated with recovery, death, retirement, renegotiation and lockdown being lifted. The expression is as follows

$$
\begin{aligned}
rW^L_{it}(w,\alpha,x;L) &= w + \rho_y\left(W^L_{rt}(w,\alpha,x,1;L)-W^L_{it}(w,\alpha,x;L)\right) \\
&+ \gamma_y(\ell_{it})(0-W^L_{it}(w,\alpha,x;L)) \\
&+ \delta\left(U^L_{it}-W^L_{it}(w,\alpha,x;L)\right) + \eta\left(R^L_{it}-W^L_{it}(w,\alpha,x;L)\right) \\
&+ \nu\left(\beta\max\{S^L_{it}(\alpha,x;L),0\}+U^L_{it}-W^L_{it}(w,\alpha,x;L)\right) \\
&+ \Lambda(W_{it}(w,\alpha,x)-W^L_{it}(w,\alpha,x;L)) + \dot{W}^L_{it}(w,\alpha,x;L)
\end{aligned}
$$

and similarly for an unlocked infected worker, the expression is

$$
\begin{aligned}
rW^L_{it}(w,\alpha,x;U) &= w + \rho_y\left(W^L_{rt}(w,\alpha,x,1;U)-W^L_{it}(w,\alpha,x;U)\right) \\
&+ \gamma_y(\ell_{it})(0-W^L_{it}(w,\alpha,x;U)) \\
&+ \delta\left(U^L_{it}-W^L_{it}(w,\alpha,x;U)\right) + \eta\left(R^L_{it}-W^L_{it}(w,\alpha,x;U)\right) \\
&+ \nu\left(\beta\max\{S^L_{it}(\alpha,x;U),0\}+U^L_{it}-W^L_{it}(w,\alpha,x;U)\right) \\
&+ \Lambda(W_{it}(w,\alpha,x)-W^L_{it}(w,\alpha,x;U)) + \dot{W}^L_{it}(w,\alpha,x;U).
\end{aligned}
$$

One point to note is that the match retains its status with regard to being in the locked or unlocked sector throughout the health status changes of the worker. That is — whatever sector their match belonged to prior and during infection — the match will remain in that sector subsequent to recovery.

Value of a filled vacancy with an infected worker

A firm that has an infected worker pays their wage as sick pay for the duration of their illness in the absence of separation. The firm receives the value associated with the possibility of the worker's recovery and the match can be broken through either exogenous or endogenous separation at re-negotiation, retirement or through death of the worker from the virus. The formal expression for a match with an infected worker in a locked sector is given by

$$\begin{aligned} rJ_{it}^L(w,\alpha,x;L) &= -w + \rho_y\left(J_{rt}^L(w,\alpha,x,1;L) - J_{it}^L(w,\alpha,x;L)\right) \\ &\quad + (\gamma_y(\ell_{it}) + \delta + \eta)(V_t^L - J_{it}^L(w,\alpha,x;L)) \\ &\quad + \nu\left((1-\beta)\max\{S_{it}^L(\alpha,x;L),0\} + V_t^L - J_{it}^L(w,\alpha,x;L)\right) \\ &\quad + \Lambda(J_{it}(w,\alpha,x) - J_{it}^L(w,\alpha,x;L)) + \dot{J}_{it}^L(w,\alpha,x;L) \end{aligned}$$

while that in the unlocked sector is

$$\begin{aligned} rJ_{it}^L(w,\alpha,x;U) &= -w + \rho_y\left(J_{rt}^L(w,\alpha,x,1;U) - J_{it}^L(w,\alpha,x;U)\right) \\ &\quad + (\gamma_y(\ell_{it}) + \delta + \eta)(V_t^L - J_{it}^L(w,\alpha,x;U)) \\ &\quad + \nu\left((1-\beta)\max\{S_{it}^L(\alpha,x;U),0\} + V_t^L - J_{it}^L(w,\alpha,x;U)\right) \\ &\quad + \Lambda(J_{it}(w,\alpha,x) - J_{it}^L(w,\alpha,x;U)) + \dot{J}_{it}^L(w,\alpha,x;U) \end{aligned}$$

Surplus of a match with an infected worker

The surplus can be found through using the value functions for employment, unemployment and value of a filled job for the locked and unlocked sectors to get the surplus. Using the equilibrium condition that the value to a vacancy is zero, the surplus for a locked match is

$$\begin{aligned} (r + \rho_y + \gamma_y(\ell_{it}) + \delta + \eta + \nu + \Lambda)S_{it}^L(\alpha,x;L) &= -b_u + \rho_y S_{rt}^L(\alpha,x;L) \\ &\quad + \nu\max\{S_{it}^L(\alpha,x;L),0\} + \Lambda S_{it}(\alpha,x) + \dot{S}_{it}^L(\alpha,x;L) \end{aligned}$$

while that for an unlocked match is

$$\begin{aligned} (r + \rho_y + \gamma_y(\ell_{it}) + \delta + \eta + \nu + \Lambda)S_{it}^L(\alpha,x;U) &= -b_u + \rho_y S_{rt}^L(\alpha,x;U) \\ &\quad + \nu\max\{S_{it}^L(\alpha,x;U),0\} + \Lambda S_{it}(\alpha,x) + \dot{S}_{it}^L(\alpha,x;U). \end{aligned}$$

Susceptible young workers

The value to being an unemployed susceptible worker closely resembles that of a recovered worker, with the exception of an additional value change associated with the possibility of infection. Their value function is given as

$$
\begin{aligned}
rU_{st}^L = b_u &+ \phi_t \pi \beta \int\int \max\{S_{st}^L(\alpha, x, 0; L), S_{st}^L(\alpha, x, 1; L), 0\} d^2F(\alpha, x) \\
&+ \phi_t(1-\pi)\beta \int\int \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} d^2F(\alpha, x) \\
&+ \lambda_0^L \ell_{it}(U_{it}^L - U_{st}^L) + \eta(R_{st}^L - U_{st}^L) + \Lambda(U_{st} - U_{st}^L) + \dot{U}_{st}^L
\end{aligned}
$$

where notice that the rate of infection is given by $\lambda_0^L \ell_{it}$, using the lockdown infection parameter that exists regardless of working decisions. A worker that is employed with contract for wages and working arrangements (w, m) has value that differs based on the sector they work in. When locked, the worker's value function is given by

$$
\begin{aligned}
rW_{st}^L(w, \alpha, x, m; L) = w &+ \delta(U_{st}^L - W_{st}^L(w, \alpha, x, m; L)) + \eta(R_{st}^L - W_{st}^L(w, \alpha, x, m; L)) \\
&+ \lambda_0^L \ell_{it} \left(W_{it}^L(w, \alpha, x, L) - W_{st}^L(w, \alpha, x, m; L)\right) \\
&+ \nu \left(\beta \max\{S_{st}^L(\alpha, x, 0; L), S_{st}^L(\alpha, x, 1; L), 0\} + U_{st}^L - W_{st}^L(w, \alpha, x, m; L)\right) \\
&+ \Lambda(W_{st}(w, \alpha, x, m) - W_{st}^L(w, \alpha, x, m; L)) + \dot{W}_{st}^L(w, \alpha, x, m; L)
\end{aligned}
$$

where notice that their infection rate is the same as that of the unemployed worker given that they are unable to work away from home. In contrast, a worker in an unlocked sector has value functions that differ based on the contracted m. For $m = 0$, see that

$$
\begin{aligned}
rW_{st}^L(w, \alpha, x, 0; U) = w &+ \delta(U_{st}^L - W_{st}^L(w, \alpha, x, 0; U)) + \eta(R_{st}^L - W_{st}^L(w, \alpha, x, 0; U)) \\
&+ \lambda_0^L \ell_{it} \left(W_{it}^L(w, \alpha, x, U) - W_{st}^L(w, \alpha, x, 0; U)\right) \\
&+ \nu \left(\beta \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} + U_{st}^L - W_{st}^L(w, \alpha, x, 0; U)\right) \\
&+ \Lambda(W_{st}(w, \alpha, x, 0) - W_{st}^L(w, \alpha, x, 0; U)) + \dot{W}_{st}^L(w, \alpha, x, 0; U)
\end{aligned}
$$

and then for $m = 1$

$$
\begin{aligned}
rW_{st}^L(w, \alpha, x, 1; U) = w &+ \delta(U_{st}^L - W_{st}^L(w, \alpha, x, 1; U)) + \eta(R_{st}^L - W_{st}^L(w, \alpha, x, 1; U)) \\
&+ \left[\lambda_0^L + \lambda_1\right] \ell_{it} \left(W_{it}^L(w, \alpha, x, U) - W_{st}^L(w, \alpha, x, 1; U)\right) \\
&+ \nu \left(\beta \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} + U_{st}^L - W_{st}^L(w, \alpha, x, 1; U)\right) \\
&+ \Lambda(W_{st}(w, \alpha, x, 0) - W_{st}^L(w, \alpha, x, 1; U)) + \dot{W}_{st}^L(w, \alpha, x, 1; U)
\end{aligned}
$$

where the distinction between the two is the higher infection rate when working away from home.

Value of a filled vacancy with an susceptible worker

The value to a filled job with a susceptible worker in a locked industry for arbitrary contract (w, m) is given as follows

$$\begin{aligned} rJ^L_{st}(w,\alpha,x,m;L) = {} & p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,m;L)) \\ & + \lambda^L_0 \ell_{it} \left(J^L_{it}(w,\alpha,x;L) - J^L_{st}(w,\alpha,x,m;L)\right) \\ & + \nu \left((1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), 0\} + V^L_t - J^L_{st}(w,\alpha,x,m;L)\right) \\ & + \Lambda(J_{st}(w,\alpha,x,m) - J^L_{st}(w,\alpha,x,m;L)) + \dot{J}_{st}(w,\alpha,x,m;L). \end{aligned}$$

A match that is unlocked differs based on locked status. An unlocked match with $m = 0$ delivers value of

$$\begin{aligned} rJ^L_{st}(w,\alpha,x,0;U) = {} & p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,0;U)) \\ & + \lambda^L_0 \ell_{it} \left(J^L_{it}(w,\alpha,x;U) - J^L_{st}(w,\alpha,x,0;U)\right) \\ & + \nu \left((1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,0;U)\right) \\ & + \Lambda(J_{st}(w,\alpha,x,0) - J^L_{st}(w,\alpha,x,0;U)) + \dot{J}_{st}(w,\alpha,x,0;U). \end{aligned}$$

and that for an $m = 1$ contract gives

$$\begin{aligned} rJ^L_{st}(w,\alpha,x,1;U) = {} & p(\alpha,x,1) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,1;U)) \\ & + [\lambda^L_0 + \lambda_1] \ell_{it} \left(J^L_{it}(w,\alpha,x;U) - J^L_{st}(w,\alpha,x,1;U)\right) \\ & + \nu \left((1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,1;U)\right) \\ & + \Lambda(J_{st}(w,\alpha,x,1) - J^L_{st}(w,\alpha,x,1;U)) + \dot{J}_{st}(w,\alpha,x,1;U). \end{aligned}$$

where the match now produces the higher away from home level of output, in addition to the rate of change to infected status being higher by λ_1.

Surplus of a match with a susceptible worker

The surplus from a match in the locked sector for $m \in \{0, 1\}$ is given as

$$\begin{aligned} (r+\delta+\eta+\lambda^L_0\ell_{it}+\nu+\Lambda)S^L_{st}(\alpha,x,m;L) = {} & p(\alpha,x,0) - b_u \\ & - \phi_t \pi \beta \int\int \max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), 0\} d^2F(\alpha,x) \\ & - \phi_t (1-\pi) \beta \int\int \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} d^2F(\alpha,x) \\ & + \lambda^L_0 \ell_{it} S^L_{it}(\alpha,x;L) + \nu \max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), 0\} \\ & + \Lambda S_{st}(\alpha,x,m) + \dot{S}^L_{st}(\alpha,x,m;L). \end{aligned}$$

The surplus for a match in the unlocked sector with $m = 0$ is

$$
\begin{aligned}
(r+\delta+\eta+\lambda_0^L\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,0;U) &= p(\alpha,x,0)-b_u \\
&- \phi_t\pi\beta\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}d^2F(\alpha,x) \\
&- \phi_t(1-\pi)\beta\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}d^2F(\alpha,x) \\
&+ \lambda_0^L\ell_{it}S_{it}^L(\alpha,x;U)+\nu\max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\} \\
&+ \Lambda S_{st}(\alpha,x,0)+\dot{S}_{st}^L(\alpha,x,0;U)
\end{aligned}
$$

and that for the unlocked sector with $m = 1$ is

$$
\begin{aligned}
(r+\delta+\eta+(\lambda_0^L+\lambda_1)\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,1;U) &= p(\alpha,x,1)-b_u \\
&- \phi_t\pi\beta\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}d^2F(\alpha,x) \\
&- \phi_t(1-\pi)\beta\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}d^2F(\alpha,x) \\
&+ (\lambda_0^L+\lambda_1)\ell_{it}S_{it}^L(\alpha,x;U)+\lambda_1\ell_{it}(U_{it}^L-U_{st}^L) \\
&+ \nu\max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\} \\
&+ \Lambda S_{st}(\alpha,x,1)+\dot{S}_{st}^L(\alpha,x,1;U).
\end{aligned}
$$

1.2 Vacant jobs

Vacant jobs get in contact with unemployed workers at a rate ϕ_t^f. Upon contracting, the firm receives fraction $(1-\beta)$ of the match's generated surplus. Notice that there are four possibilities for a given vacancy with regard to the type of match that is formed. The potential unemployed workers they can match with differ along the health dimension — they could either be susceptible or recovered. In addition, there are two possibilities from the perspective of the production being in either the locked or unlocked sectors. As such, there are five terms in the value to a vacancy: the flow cost κ as well as terms capturing these four possibilities

$$
\begin{aligned}
rV_t^L = -\kappa &+ \phi_t^f(1-\beta)\frac{u_{st}^L}{u_{st}^L+u_{rt}^L}\pi\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}dF^2(\alpha,x) \\
&+ \phi_t^f(1-\beta)\frac{u_{st}^L}{u_{st}^L+u_{rt}^L}(1-\pi)\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}dF^2(\alpha,x) \\
&+ \phi_t^f(1-\beta)\frac{u_{rt}^L}{u_{st}^L+u_{rt}^L}\pi\int\int \max\{S_{rt}^L(\alpha,x;L),0\}dF^2(\alpha,x) \\
&+ \phi_t^f(1-\beta)\frac{u_{rt}^L}{u_{st}^L+u_{rt}^L}(1-\pi)\int\int \max\{S_{rt}^L(\alpha,x;U),0\}dF^2(\alpha,x)
\end{aligned}
$$

where u_{st}^L and u_{rt}^L denote the measures of unemployed workers with susceptible and recovered status under the lockdown model.

2 Dynamics of the lockdown model

This section details the dynamics of the measures of workers in differing employment, age and health statuses in the model with lockdown. Measures with L superscripts correspond to those under lockdown, while those without are from the baseline model, (out of lockdown). Assume that the lockdown commences at an arbitrary time denoted by $\bar{t}$. At this point, measures are divided-up as follows. The measures of the three different health statuses for unemployment will be equal to their pre-lockdown measures

$$
\begin{aligned}
u_{s\bar{t}}^L &= u_{s\bar{t}} \\
u_{i\bar{t}}^L &= u_{i\bar{t}} \\
u_{r\bar{t}}^L &= u_{r\bar{t}}
\end{aligned}
$$

which are the measures of susceptible, infected and recovered respectively. Similarly for the retired at the time of lockdown's commencement

$$
\begin{aligned}
o_{s\bar{t}}^L &= o_{s\bar{t}} \\
o_{i\bar{t}}^L &= o_{i\bar{t}} \\
o_{r\bar{t}} &= o_{r\bar{t}}
\end{aligned}
$$

which are the measures of retired people across the susceptible, infected and recovered health statuses respectively. The measures of employed workers of a given health status and match state (α, x) will be split such that fraction π will be placed into the locked sector while fraction $1-\pi$ will be in the unlocked sector as follows

$$\begin{aligned}
e^L_{0s\bar{t}}(\alpha, x; L) &= \pi e_{0s\bar{t}}(\alpha, x) \\
e^L_{0s\bar{t}}(\alpha, x; U) &= (1-\pi) e_{0s\bar{t}}(\alpha, x) \\
e^L_{1s\bar{t}}(\alpha, x; L) &= \pi e_{1s\bar{t}}(\alpha, x) \\
e^L_{1s\bar{t}}(\alpha, x; U) &= (1-\pi) e_{1s\bar{t}}(\alpha, x) \\
e^L_{r\bar{t}}(\alpha, x; L) &= \pi e_{r\bar{t}}(\alpha, x) \\
e^L_{r\bar{t}}(\alpha, x; U) &= (1-\pi) e_{r\bar{t}}(\alpha, x) \\
e^L_{i\bar{t}}(\alpha, x; L) &= \pi e_{i\bar{t}}(\alpha, x) \\
e^L_{i\bar{t}}(\alpha, x; U) &= (1-\pi) e_{i\bar{t}}(\alpha, x)
\end{aligned}$$

which respectively represent the measures of susceptible workers at home in locked jobs and unlocked jobs, of susceptible workers away from home in locked and unlocked jobs, of recovered workers in locked and unlocked jobs and infected workers in locked and unlocked jobs. From the point where the lockdown commences, these measures evolve endogenously. The laws of motion for the retired are

$$\begin{aligned}
\dot{o}^L_{st} &= \eta \left(u^L_{st} + \int\int e^L_{0st}(\alpha, x; L) d\alpha dx + \int\int e^L_{0st}(\alpha, x; U) d\alpha dx \right) \\
&\quad + \eta \left(\int\int e^L_{1st}(\alpha, x; L) d\alpha dx + \int\int e^L_{1st}(\alpha, x; U) d\alpha dx \right) - (\lambda^L_0 \ell^L_{it} + \chi) o^L_{st}
\end{aligned}$$

$$\begin{aligned}
\dot{o}^L_{it} &= \eta \left(u^L_{it} + \int\int e^L_{it}(\alpha, x; L) d\alpha dx + \int\int e^L_{it}(\alpha, x; U) d\alpha dx \right) \\
&\quad + \lambda^L_0 \ell^L_{it} o^L_{st} - (\gamma_o(\ell^L_{it}) + \chi + \rho_o) o^L_{it}
\end{aligned}$$

$$\begin{aligned}
\dot{o}^L_{rt} &= \eta \left(u^L_{rt} + \int\int e^L_{rt}(\alpha, x; L) d\alpha dx + \int\int e^L_{rt}(\alpha, x; U) d\alpha dx \right) \\
&\quad + \rho_o o^L_{it} - \chi o^L_{rt}
\end{aligned}$$

while those for the unemployed are

$$
\begin{aligned}
\dot{u}^L_{st} &= \psi + \delta \int\int e^L_{0st}(\alpha, x; L) d\alpha dx + \delta \int\int e^L_{1st}(\alpha, x; L) d\alpha dx \\
&+ \delta \int\int e^L_{0st}(\alpha, x; U) d\alpha dx + \delta \int\int e^L_{1st}(\alpha, x; U) d\alpha dx \\
&+ \nu \int\int e^L_{0st}(\alpha, x; L)\{S^L_{0st}(\alpha, x; L) < 0\} d\alpha dx + \nu \int\int e^L_{1st}(\alpha, x; L)\{S^L_{1st}(\alpha, x; L) < 0\} d\alpha dx \\
&+ \nu \int\int e^L_{0st}(\alpha, x; U)\{S^L_{0st}(\alpha, x; U) < 0\} d\alpha dx + \nu \int\int e^L_{1st}(\alpha, x; U)\{S^L_{1st}(\alpha, x; U) < 0\} d\alpha dx \\
&- \phi^L_t \pi \int\int \{S^L_{st}(\alpha, x, L) \geq 0\} d^2F(\alpha, x) u^L_{st} - \phi^L_t (1-\pi) \int\int \{S^L_{st}(\alpha, x, U) \geq 0\} d^2F(\alpha, x) u^L_{st} \\
&- \lambda^L_0 \ell^L_{it} u^L_{st} - \eta u^L_{st}
\end{aligned}
$$

$$
\begin{aligned}
\dot{u}^L_{it} &= \delta \int\int e^L_{it}(\alpha, x; L) d\alpha dx + \delta \int\int e^L_{it}(\alpha, x; U) d\alpha dx \\
&+ \nu \int\int e^L_{it}(\alpha, x; L)\{S^L_{it}(\alpha, x; L) < 0\} d\alpha dx + \nu \int\int e^L_{it}(\alpha, x; U)\{S^L_{it}(\alpha, x; U) < 0\} d\alpha dx \\
&+ \lambda^L_0 \ell^L_{it} u^L_{st} - (\rho_y + \gamma(\ell^L_{it}) + \eta) u^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
\dot{u}^L_{rt} &= \delta \int\int e^L_{rt}(\alpha, x; L) d\alpha dx + \delta \int\int e^L_{rt}(\alpha, x; U) d\alpha dx \\
&+ \nu \int\int e^L_{rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) < 0\} d\alpha dx + \nu \int\int e^L_{rt}(\alpha, x; U)\{S^L_{rt}(\alpha, x; U) < 0\} d\alpha dx \\
&+ \rho_y u^L_{it} - \phi^L_t \pi \int\int \{S^L_{rt}(\alpha, x, L) \geq 0\} d^2F(\alpha, x) u^L_{rt} - \phi^L_t (1-\pi) \int\int \{S^L_{rt}(\alpha, x, U) \geq 0\} d^2F(\alpha, x) u^L_{rt}.
\end{aligned}
$$

For the employed workers, we track measures across the different match characteristics and lock statuses. For measures of susceptible employed, the equations are as follows.

$$\begin{aligned}\dot{e}^L_{0st}(\alpha,x;L) &= \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}f(\alpha,x)\\ &-(\delta+\eta)e^L_{0st}(\alpha,x;L)-\nu e^L_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x,0;L)<0\}\\ &-\nu e^L_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}\\ &+\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}\\ &-e^L_{0st}(\alpha,x;L)\lambda^L_0\ell^L_{it}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{0st}(\alpha,x;U) &= (1-\pi)u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\ &-(\delta+\eta)e^L_{0st}(\alpha,x;U)-\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x,0;U)<0\}\\ &-\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}\\ &+\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}\\ &-e^L_{0st}(\alpha,x;U)\lambda^L_0\ell^L_{it}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{1st}(\alpha,x;L) &= \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}f(\alpha,x)\\ &-(\delta+\eta)e^L_{1st}(\alpha,x;L)-\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x,1;L)<0\}\\ &-\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}\\ &+\nu e_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}\\ &-e^L_{1st}(\alpha,x;L)\lambda^L_0\ell^L_{it}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{1st}(\alpha,x;U) &= (1-\pi)u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\ &-(\delta+\eta)e^L_{1st}(\alpha,x;U)-\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x,1;U)<0\}\\ &-\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}\\ &+\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}\\ &-e^L_{1st}(\alpha,x;U)(\lambda^L_0+\lambda_1)\ell^L_{it}.\end{aligned}$$

For the measures of infected employed, the dynamics evolve according to

$$\begin{aligned}\dot{e}^L_{it}(\alpha,x;L) &= e^L_{0st}(\alpha,x;L)\lambda^L_0\ell^L_{it}+e^L_{1st}(\alpha,x;L)(\lambda^L_0)\ell^L_{it}-\nu e^L_{it}(\alpha,x;L)\{S^L_{it}(\alpha,x;L)<0\}\\ &-(\delta+\rho_y+\gamma(\ell^L_{it})+\eta)e^L_{it}(\alpha,x;L)\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{it}(\alpha,x;U) &= e^L_{0st}(\alpha,x;U)\lambda^L_0\ell^L_{it}+e^L_{1st}(\alpha,x;U)(\lambda^L_0+\lambda_1)\ell^L_{it}-\nu e^L_{it}(\alpha,x;U)\{S^L_{it}(\alpha,x;U)<0\}\\ &-(\delta+\rho_y+\gamma(\ell^L_{it})+\eta)e^L_{it}(\alpha,x;U).\end{aligned}$$

The measures of employed that are recovered evolve as follows

$$\begin{aligned}\dot{e}^L_{rt}(\alpha, x; L) &= \pi u^L_{rt}\phi_t\{S^L_{rt}(\alpha, x; L) \geq 0\}f(\alpha, x) + \rho_y e^L_{it}(\alpha, x; L) - (\delta + \eta)e^L_{rt}(\alpha, x; L) \\ &\quad - \nu e^L_{rt}(\alpha, x)\{S^L_{rt}(\alpha, x; L) < 0\}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{rt}(\alpha, x; U) &= (1 - \pi)u^L_{rt}\phi^L_t\{S^L_{rt}(\alpha, x; U) \geq 0\}f(\alpha, x) + \rho_y e^L_{it}(\alpha, x; U) - (\delta + \eta)e^L_{rt}(\alpha, x; U) \\ &\quad - \nu e^L_{rt}(\alpha, x)\{S^L_{rt}(\alpha, x; U) < 0\}.\end{aligned}$$

Finally, denote the time where lockdown ceases by $\hat{t}$. At this time the measure of unemployed of each status for non-lockdown are equal to their lockdown level as follows

$$\begin{aligned}u_{s\hat{t}} &= u^L_{s\hat{t}} \\ u_{i\hat{t}} &= u^L_{i\hat{t}} \\ u_{r\hat{t}} &= u^L_{r\hat{t}}\end{aligned}$$

for susceptible, infected and retired respectively. Similarly, for the three health statuses of retired workers

$$\begin{aligned}o_{s\hat{t}} &= o^L_{s\hat{t}} \\ o_{i\hat{t}} &= o^L_{i\hat{t}} \\ o_{r\hat{t}} &= o^L_{r\hat{t}}.\end{aligned}$$

Lastly, for each employment health status and idiosyncratic match state, the measures of locked and unlocked matches are summed to together as follows

$$\begin{aligned}e_{0s\hat{t}}(\alpha, x) &= e^L_{0s\hat{t}}(\alpha, x; L) + e^L_{0s\hat{t}}(\alpha, x; U) \\ e_{1s\hat{t}}(\alpha, x) &= e^L_{1s\hat{t}}(\alpha, x; L) + e^L_{1s\hat{t}}(\alpha, x; U) \\ e_{r\hat{t}}(\alpha, x) &= e^L_{r\hat{t}}(\alpha, x; L) + e^L_{r\hat{t}}(\alpha, x; U) \\ e_{i\hat{t}}(\alpha, x) &= e^L_{i\hat{t}}(\alpha, x; L) + e^L_{i\hat{t}}(\alpha, x; U)\end{aligned}$$

which are for the susceptible employed working at home and away from home, the recovered and infected respectively. From time $\hat{t}$ onwards, the measures evolve endogenously as described in appendix A.2 in the main text.

3 Wages under laissez-faire

Recovered individuals

For a recovered individual, an arbitrary wage leaves the employer with value equal to:

$$\begin{aligned}(r+\delta+\eta)J_{rt}(w,\alpha,x) &= p(\alpha,x,1)-w \\ &\quad + \nu\left((1-\beta)\max\{S_{rt}(\alpha,x),0\} - J_{rt}(w,\alpha,x)\right) + \dot{J}_{rt}(w,\alpha,x)\end{aligned}$$

Re-arranging terms, and substituting in the Nash splitting condition, $J_{rt}(w,\alpha,x) = (1-\beta)S_{rt}(\alpha,x)$, we get the following wage function for recovered employees:

$$\begin{aligned}w_{rt}(\alpha,x) &= p(\alpha,x,1) + \nu(1-\beta)\max\{S_{rt}(\alpha,x),0\} \\ &\quad - (r+\delta+\eta+\nu)(1-\beta)S_{rt}(\alpha,x) + (1-\beta)\dot{S}_{rt}(\alpha,x)\end{aligned}$$

Infected individuals

The infected never start new jobs (by assumption) and inherent their previous wages and are on sick leave. However, they can be hit by re-negotiation shock. Again for arbitrary wage,

$$\begin{aligned}rJ_{it}(w,\alpha,x) &= -w + \rho\left(J_{rt}(w,\alpha,x) - J_{it}(w,\alpha,x)\right) + (\gamma(\ell_{it})+\delta+\eta)(-J_{it}(w,\alpha,x)) \\ &\quad + \nu\left((1-\beta)\max\{S_{it}(\alpha,x),0\} - J_{it}(w,\alpha,x)\right) + \dot{J}_{it}(w,\alpha,x)\end{aligned}$$

re-arranging terms, and substituting again in the Nash splitting condition, $J_{it}(w,\alpha,x) = (1-\beta)S_{it}(\alpha,x)$, we obtain:

$$\begin{aligned}w_{it}(\alpha,x) &= \rho J_{rt}(w,\alpha,x) + \nu(1-\beta)\max\{S_{it}(\alpha,x),0\} \\ &\quad - (r+\rho+\gamma(\ell_{it})+\delta+\eta+\nu)(1-\beta)S_{it}(w,\alpha,x) + (1-\beta)\dot{S}_{it}(w,\alpha,x)\end{aligned}$$

There is a unique solution as derivative of lhs is negative. But it could be negative if component independent of wage is negative (and large). Since:

$$\begin{aligned}(r+\delta+\eta+\nu)J_{rt}(w_{it}(\alpha,x),\alpha,x) &= p(\alpha,x,1) - w_{it}(\alpha,x) \\ &\quad + \nu(1-\beta)\max\{S_{rt}(\alpha,x),0\} + \dot{J}_{rt}(w_{it}(\alpha,x),\alpha,x)\end{aligned}$$

then substituting:

$$\begin{aligned}w_{it}(\alpha,x)\frac{r+\delta+\eta+\nu+\rho}{r+\delta+\eta+\nu} &= \frac{\rho}{r+\delta+\eta+\nu}p(\alpha,x,1) + \frac{\rho\nu(1-\beta)}{r+\delta+\eta+\nu}\max\{S_{rt}(\alpha,x),0\} \\ &\quad + \frac{\rho}{r+\delta+\eta+\nu}\dot{J}_{rt}(w_{it}(\alpha,x),\alpha,x) \\ &\quad - (r+\rho+\gamma(\ell_{it})+\delta+\eta)(1-\beta)S_{it}(\alpha,x) + (1-\beta)\dot{S}_{it}(\alpha,x)\end{aligned}$$

Re-arranging terms, and using the Nash splitting condition, $J_{rt}(w, \alpha, x) = (1 - \beta)S_{rt}(\alpha, x)$, we get a final expression for the wage of infected employees:

$$
\begin{aligned}
w_{it}(\alpha, x) = & \frac{\rho}{r + \delta + \eta + \nu + \rho} p(\alpha, x, 1) + \frac{\rho\nu(1 - \beta)}{r + \delta + \eta + \nu + \rho} \max\{S_{rt}(\alpha, x), 0\} \\
& - \frac{(r + \delta + \eta + \nu)(r + \rho + \gamma(\ell_{it}) + \delta + \eta)}{r + \delta + \eta + \nu + \rho}(1 - \beta)S_{it}(\alpha, x) \\
& + \frac{\rho}{r + \delta + \eta + \nu + \rho}(1 - \beta)\dot{S}_{rt}(\alpha, x) \\
& + \frac{r + \delta + \eta + \nu}{r + \delta + \eta + \nu + \rho}(1 - \beta)\dot{S}_{it}(\alpha, x)
\end{aligned}
$$

Susceptible individuals

For arbitrary wage w, the value of a job (α, x) being worked from home by a susceptible individual is equal to:

$$
\begin{aligned}
(r + \delta + \eta + \lambda_0\ell_{it} + \nu)J_{st}(w, \alpha, x, 0) = & \, p(\alpha, x, 0) - w + \lambda_0\ell_{it}J_{it}(w, \alpha, x) \\
& + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\}) + \dot{J}_{st}(w, \alpha, x, 0)
\end{aligned}
$$

Re-arranging terms and using the Nash splitting conditions $J_{st}(w, \alpha, x, 0) = (1-\beta)S_{st}(w, \alpha, x, 0)$ and $J_{it}(w, \alpha, x) = (1 - \beta)S_{it}(w, \alpha, x)$, we can write the wage equation for susceptible employees only working at home as follows:

$$
\begin{aligned}
w_{st}(\alpha, x, 0) = & \, p(\alpha, x, 0) + \lambda_0\ell_{it}(1 - \beta)S_{it}(\alpha, x) \\
& - (r + \delta + \eta + \lambda_0\ell_{it} + \nu)(1 - \beta)S_{st}(\alpha, x, 0) \\
& + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\}) + (1 - \beta)\dot{S}_{st}(\alpha, x, 0)
\end{aligned}
$$

Similarly, the wage equation for susceptible employees working away from home is equal to:

$$
\begin{aligned}
w_{st}(\alpha, x, 1) = & \, p(\alpha, x, 1) + (\lambda_0 + \lambda_1)\ell_{it}(1 - \beta)S_{it}(\alpha, x) \\
& - (r + \delta + \eta + (\lambda_0 + \lambda_1)\ell_{it} + \nu)(1 - \beta)S_{st}(\alpha, x, 1) \\
& + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\}) + (1 - \beta)\dot{S}_{st}(\alpha, x, 1)
\end{aligned}
$$

4 Wages under lockdown

Recovered individuals

During lockdown, the job value for a match that is unlocked with a recovered individual at an arbitrary wage w is equal to:

$$\begin{aligned}(r+\delta+\eta+\nu+\Lambda)J_{rt}^{L}(w,\alpha,x;U) = p(\alpha,x,1) - w + \nu(1-\beta)\max\{S_{rt}^{L}(\alpha,x;U),0\}\\ + \Lambda J_{rt}(w,\alpha,x) + \dot{J}_{rt}^{L}(w,\alpha,x;U)\end{aligned}$$

Re-arranging terms and substituting in the Nash splitting condition, $J_{rt}^{L}(w,\alpha,x;U) = (1-\beta)S_{rt}^{L}(\alpha,x;U)$, the wage for a recovered individual in an unlocked job during lockdown takes the following form:

$$\begin{aligned}w_{rt}(\alpha,x;U) = p(\alpha,x,1) + \nu(1-\beta)\max\{S_{rt}^{L}(\alpha,x;U),0\} - (r+\delta+\eta+\nu+\Lambda)(1-\beta)S_{rt}^{L}(\alpha,x;U)\\ + \Lambda(1-\beta)S_{rt}(\alpha,x) + (1-\beta)\dot{S}_{rt}^{L}(\alpha,x;U)\end{aligned}$$

Similarly, the wage equation for a recovered individual in a locked job is equal to:

$$\begin{aligned}w_{rt}(\alpha,x;L) = p(\alpha,x,0) + \nu(1-\beta)\max\{S_{rt}^{L}(\alpha,x;L),0\} - (r+\delta+\eta+\nu+\Lambda)(1-\beta)S_{rt}^{L}(\alpha,x;L)\\ + \Lambda(1-\beta)S_{rt}(\alpha,x) + (1-\beta)\dot{S}_{rt}^{L}(\alpha,x;L)\end{aligned}$$

Infected individuals

The infected never start new jobs (by assumption) and are on sick leave. They inherit their previous wages and their lockdown status. However, they can be hit by re-negotiation shock. Given the job value for a locked match with an infected worker at an arbitrary wage w, we can write:

$$\begin{aligned}w = \rho_y J_{rt}^{L}(w,\alpha,x;L) + \nu(1-\beta)\max\{S_{it}^{L}(\alpha,x;L),0\}\\ + \Lambda J_{it}(w,\alpha,x) + \dot{J}_{it}^{L}(w,\alpha,x;L) - (r+\rho_y+\gamma_y(\ell_{it})+\delta+\eta+\nu+\Lambda)J_{it}^{L}(w,\alpha,x;L)\end{aligned}$$

Substituting again the Nash splitting condition $J_{it}^{L}(w,\alpha,x;L) = (1-\beta)S_{it}^{L}(w,\alpha,x;L)$, we get the following wage equation for an infected worker in a locked match:

$$\begin{aligned}w_{it}(\alpha,x;L) = \rho_y(1-\beta)S_{rt}^{L}(\alpha,x;L) + \nu(1-\beta)\max\{S_{it}^{L}(\alpha,x;L),0\} + \Lambda(1-\beta)S_{it}(\alpha,x)\\ - (r+\rho_y+\gamma_y(\ell_{it})+\delta+\eta+\Lambda)(1-\beta)S_{it}^{L}(\alpha,x;L) + (1-\beta)\dot{S}_{it}^{L}(\alpha,x;L)\end{aligned}$$

Similarly, the wage equation for an infected worker in an unlocked match during lockdown is equal to:

$$\begin{aligned}w_{it}(\alpha,x;U) = \rho_y(1-\beta)S_{rt}^{L}(\alpha,x;U) + \nu(1-\beta)\max\{S_{it}^{L}(\alpha,x;U),0\} + \Lambda(1-\beta)S_{it}(\alpha,x)\\ - (r+\rho_y+\gamma_y(\ell_{it})+\delta+\eta+\Lambda)(1-\beta)S_{it}^{L}(\alpha,x;U) + (1-\beta)\dot{S}_{it}^{L}(\alpha,x;U)\end{aligned}$$

Susceptible individuals

An arbitrary wage w to a susceptible individual leaves the employer in a locked match with the following value:

$$
\begin{aligned}
rJ^L_{st}(w,\alpha,x,m;L) = {} & p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,m;L)) \\
& + \lambda^L_{0y}\ell_{it}\left(J^L_{it}(w,\alpha,x;L) - J^L_{st}(w,\alpha,x,m;L)\right) \\
& + \nu\left((1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L),0\} + V^L_t - J^L_{st}(w,\alpha,x,m;L)\right) \\
& + \Lambda(J_{st}(w,\alpha,x,m) - J^L_{st}(w,\alpha,x,m;L)) + \dot{J}_{st}(w,\alpha,x,m;L)
\end{aligned}
$$

for $m \in \{0,1\}$. Imposing free entry $V^L_t = 0$, and re-arranging terms, we get:

$$
\begin{aligned}
w = {} & p(\alpha,x,0) - (r+\delta+\eta+\lambda^L_{0y}\ell_{it}+\nu+\Lambda)J^L_{st}(w,\alpha,x,m;L) \\
& + \lambda^L_{0y}\ell_{it}J^L_{it}(w,\alpha,x;L) \\
& + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L),0\} \\
& + \Lambda J_{st}(w,\alpha,x,m) + \dot{J}_{st}(w,\alpha,x,m;L)
\end{aligned}
$$

Substituting the Nash splitting rule $J^L_{st}(w,\alpha,x,m;L) = (1-\beta)S^L_{st}(\alpha,x,m;L)$, the wage for a susceptible worker with a contractual $m \in \{0,1\}$ in a locked match can be written as follows:

$$
\begin{aligned}
w_{st}(\alpha,x,m;L) = {} & p(\alpha,x,0) - (r+\delta+\eta+\lambda^L_{0y}\ell_{it}+\nu+\Lambda)(1-\beta)S^L_{st}(\alpha,x,m;L) \\
& + \lambda^L_{0y}\ell_{it}(1-\beta)S^L_{it}(\alpha,x;L) \\
& + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L),0\} \\
& + \Lambda(1-\beta)S_{st}(\alpha,x,m) + (1-\beta)\dot{S}_{st}(\alpha,x,m;L)
\end{aligned}
$$

Consider now an unlocked match. An arbitrary wage w to a susceptible individual leaves the employer with the following value:

$$
\begin{aligned}
(r+\delta+\eta+\lambda^L_{0y}\ell_{it}+\nu+\Lambda)J^L_{st}(w,\alpha,x,0;U) = {} & p(\alpha,x,0) - w + \lambda^L_{0y}\ell_{it}J^L_{it}(w,\alpha,x;U) \\
& + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U),0\} \\
& + \Lambda J_{st}(w,\alpha,x,0) + \dot{J}_{st}(w,\alpha,x,0;U)
\end{aligned}
$$

when $m = 0$, and with a value equal to:

$$
\begin{aligned}
(r+\delta+\eta+(\lambda^L_{0y}+\lambda_{1y})\ell_{it}+\nu+\Lambda)J^L_{st}(w,\alpha,x,1;U) = {} & p(\alpha,x,1) - w \\
& + (\lambda^L_{0y}+\lambda_{1y})\ell_{it}J^L_{it}(w,\alpha,x;U) \\
& + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U),0\} \\
& + \Lambda J_{st}(w,\alpha,x,1) + \dot{J}_{st}(w,\alpha,x,1;U)
\end{aligned}
$$

when $m = 1$. Again, imposing free entry $V_t^L = 0$, substituting the Nash splitting rule $J_{st}^L(w, \alpha, x, m; U) = (1-\beta)S_{st}^L(\alpha, x, m; U)$ and re-arranging terms, we get a wage equation for a susceptible worker in an unlocked match working only from home during lockdown equal to:

$$\begin{aligned} w_{st}(\alpha, x, 0; U) = p(\alpha, x, 0) &+ \lambda_{0y}^L \ell_{it}(1-\beta)S_{it}^L(\alpha, x; U) \\ &- (r + \delta + \eta + \lambda_{0y}^L \ell_{it} + \nu + \Lambda)(1-\beta)S_{st}^L(\alpha, x, 0; U) \\ &+ \nu(1-\beta)\max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} \\ &+ \Lambda(1-\beta)S_{st}(\alpha, x, 0) + (1-\beta)\dot{S}_{st}(\alpha, x, 0; U) \end{aligned}$$

and a wage equation for a susceptible worker in an unlocked match working also away from home during lockdown equal to:

$$\begin{aligned} w_{st}(\alpha, x, 1; U) = p(\alpha, x, 1) &+ (\lambda_{0y}^L + \lambda_{1y})\ell_{it}(1-\beta)S_{it}^L(\alpha, x; U) \\ &- (r + \delta + \eta + (\lambda_{0y}^L + \lambda_{1y})\ell_{it} + \nu + \Lambda)(1-\beta)S_{st}^L(\alpha, x, 1; U) \\ &+ \nu(1-\beta)\max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} \\ &+ \Lambda(1-\beta)S_{st}(\alpha, x, 1) + (1-\beta)\dot{S}_{st}(\alpha, x, 1; U) \end{aligned}$$

References

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