Twin Peaks: Covid-19 and the Labor Market*

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Abstract

This paper develops a choice-theoretic equilibrium model of the labor market in the presence of a pandemic. It includes heterogeneity in productivity, age and the ability to work at home. Worker and firm behavior changes in the presence of the virus, which itself has equilibrium consequences for the infection rate. The model is calibrated to the UK and counterfactual lockdown measures are evaluated. We find a different response in both the evolution of the virus and the labor market with different degrees of severity of lockdown. We use these insights to make a labor market policy prescription to be used in conjunction with lockdown measures. Finally we find that, while the pandemic and ensuing policies impact the majority of the population negatively, consistent with recent studies, the costs are not borne equally. While the old face the highest health risks, it is the young low wage workers who suffer the most income and employment risk.

JEL Classification: I1, I3, J0, J6

1 Introduction

The COVID-19 outbreak has posed significant global challenges to public health and the economy. Since the first cases of infection reported in China in January 2020, there have been more than 7 million cases reported worldwide and at the time of writing, it has killed more than 400 thousand people. In the United Kingdom (UK), this virus has caused the death of more than 40 thousand people, with a daily peak of 1100 deaths suffered on April 21, 2020 (Figure 1). Economically, the FTSE 100 fell by 25% in the first three months of 2020, the largest quarterly fall in over three decades. Workers in the economy were particularly hard hit, with the Department of Work and Pensions processing more than five times the typical level of benefit claimants (see the second panel of Figure 1). Public lockdown policy,

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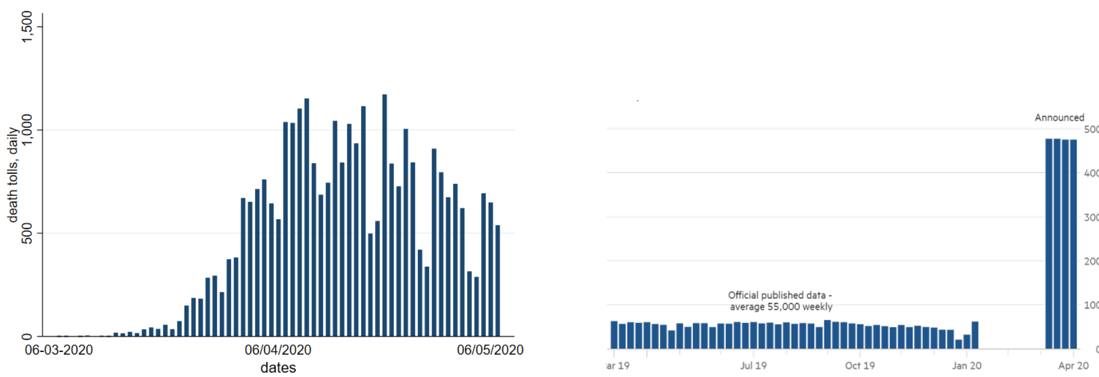
aimed at reducing the spread of the infection and ultimately saving lives, further exacerbates the economic costs associated with the pandemic.

This paper merges two workhorse models from epidemiology and economics to garner a deeper understanding of the interaction between the health and economic costs associated with the pandemic. Using the UK as a case study, we examine the implications of different lockdown policies on fatalities and the economy. Finally, we study active labor market policy that can run in conjunction with a lockdown that we argue will mitigate the economic costs.

Figure 1: The UK's Health and Economic Cost of Covid-19

Daily Deaths | Universal Credit Claims



Source: UK NHS (See https://coronavirus.data.gov.uk/) | Source: UK Department for Work and Pensions

We analyse lockdown policies of differing durations. Unsurprisingly, we find that longer lockdowns will save more lives, while causing greater harm to the economy. However, the disease and labor market dynamics can contrast quite markedly across these differing durations. In terms of the labor market, when locked down, a worker-firm pair's production falls. A longer lockdown makes it more expensive for firms to hold onto their employees. Consequently, we see a larger number of layoffs at the start of lockdown and a greater adjustment in worker-firm allocation throughout its duration. This results in lower output losses per period during the lockdown, as the market has adjusted to the new *normal*. However, it also comes with a much slower recovery. In contrast, if the lockdown is short, firms hoard their workers in anticipation of the policy's conclusion, thereby taking short term losses during the economic lockdown. This motivates our primary policy prescription — one that allows reallocation of workers during the lockdown to mitigate economic losses throughout its duration. This approach preserves the match capital before lockdown, while simultaneously allowing for a faster recovery post lockdown. The specific policy we discuss is a furlough worker scheme, in which furloughed workers can look for new employment without foregoing the government paid furloughed wage.

Our model incorporates the SIR model of infectious diseases (Kermack and McKendrick

(1927)) with the Diamond-Mortensen-Pissarides (DMP) model of the labor market (Diamond (1982), Pissarides (1985) and Mortensen and Pissarides (1994)). Fundamentally it is through these two classes of models that we can simultaneously examine the tradeoff in the health and economic cost of the pandemic. In order to understand the particularities of the Covid-19 pandemic we add three sources of heterogeneity not present in the prototypical versions of either class of model. Looking at any country the most striking feature regarding the composition of fatalities is age — Covid-19 is far more dangerous for the old than the young. Consequently, we incorporate ageing into the model. From the epidemiology perspective that means higher mortality rates for the old. Using data on fatality rates we calibrate a mortality rate for the over 65s to be twenty times larger than for those under 65s. From a labor market perspective that means retirement. A second feature of the Covid pandemic that is becoming clear is economic losses are not borne equally by workers. Those in low wage jobs face far greater income and employment risk than those in high wage jobs, (for the UK context see Adams-Prassl et al. (2020)). To this end we introduce wage dispersion into the model through an exogenous productivity distribution.

The final refinement we make is in order to link the two classes of models directly, other than through general equilibrium effects. To this end we introduce a production function that depends on, in addition to the inherent productivity of the match, the fraction of tasks that can be performed at home. While spending more time working away from home can increase total production, in a pandemic it will also increase a worker's exposure to the virus. Susceptible workers who are very productive from home, thereby foregoing little production and little of their wage, will choose to do so when the infection rate is high, slowing the spread of the pandemic. However, not all workers are afforded this luxury and as will be shown these less *lucky* workers tend to be in low paid work. Further, while workers, even in the absence of lockdown policy, will work more at home, they will do so out of self interest. When making this decision however, they do not internalize the negative externality of becoming infected on increasing the infection rate for society as a whole. This market failure additionally motivates the need for government intervention in locking down a section of the economy.

Related literature. Before the Covid-19 pandemic there existed a small theoretical literature which merged economic behavior to epidemiology models. In a standard model of disease transmission the '*basic reproduction rate*' is a constant — that is the average number of people one will infect given that the rest of the population is susceptible. In some sense the theoretical economic literature attempts to endogenize this rate. For a variety of mechanisms and diseases see, Kremer (1996), Quercioli and Smith (2006), Toxvaerd (2019, 2020) and Galeotti and Rogers (2013). In the context of our model the reproduction number depends on the decisions of how much to work away from home made by the susceptible employed. This paper is quantitative in nature and incorporates heterogeneity in many dimensions. Again, there is a small literature before this pandemic on calibrating and simulating a quantitative model of economic agents in an epidemiological framework. For the HIV virus see Greenwood et al. (2017, 2019) and Chan et al. (2016) and for Bird-flu (and now Covid) Keppo et al. (2020).

Since the outbreak of the Covid-19 pandemic there is a large and expanding number of papers building on the work of the aforementioned authors. That said, to our knowledge there is only Kapicků and Rupert (2020) that also explore how a frictional labor market interacts with a pandemic. However, the focus and exposition of their paper is quite different. A worker's health status segments the labor market and is the only source of heterogeneity. Interestingly there are papers that have leaned on the two building blocks of our model to understand disease spread, see Farboodi et al. (2020) and Garibaldi et al. (2020). But neither paper explicitly models the labor market. More broadly, there are a number of quantitative models that evaluate the economic and health tradeoffs of the pandemic and policies. Eichenbaum et al. (2020) merge the SIR model with a neo-classical representative agent model. We argue that heterogeneity is an important factor in the pandemic and our model allows for health and economic costs to vary by age, wage and occupation. Kaplan et al. (2020) account for dispersion in occupation and assets and Brotherhood et al. (2020), Favero et al. (2020) and Glover et al. (2020) use a multi-risk SIR model to account for differential mortality by age.

The rest of the paper proceeds as follows. In section 2 we setup our baseline model of the labor market and the pandemic. In section 3 we explain the role of lockdown policy. The model is calibrated to data and policy simulations are run in section 4. Section 5 concludes.

2 The baseline model

The Environment

Time is continuous and initially the economy is populated by a unit mass of individuals who are risk neutral, either young or old and discount the future at a constant rate r. Young individuals are part of the labor force and age stochastically at a Poisson rate η. A constant exogenous flow ψ of young individuals are born into unemployment. Given their age and health status workers are ex ante homogeneous and if young are ex post heterogeneous in their employment status. They can be either employed and vary in their wage w or unemployed, sustaining themselves with an exogenous flow b_u. We do not distinguish between the unemployed and the inactive and will therefore use the terms *not employed* and *unemployed* interchangeably. Old individuals are retired, they sustain themselves with exogenous flow b_o and die stochastically of natural causes at Poisson rate χ.[1] In addition to age and and labor force status individuals are characterized by a health state, h, which can be either susceptible $h = s$, infected $h = i$, or recovered $h = r$.

[1]To fix the initial population to one, the parameter ψ is set accordingly as $\psi := \frac{\eta\chi}{\eta+\chi}$.

Production

A match between a worker and a firm is characterized by two indices. A productivity index x and a technology index α, where $x, \alpha \in [0, 1]$. The variable α describes the efficiency of home working relative to working away from home. The function $h(\alpha)$ describes the measure of tasks associated with a job that can be performed at home, where $h : [0, 1] \to [0, 1]$ and $h'(\alpha) > 0$. The function $g(x)$ describes the total potential output of the worker-firm pair, where $g'(x) > 0$ and $g : [0, 1] \to \mathbb{R}^+$. We assume an ϵ cost in performing a task away from home. So any task that can be completed at home will be. Total output of a match is given by $p(\alpha, x, m)$ where $m \in \{0, 1\}$, taking the value 0 if a worker exclusively works from home and one if they ever work away from home.

$$p(\alpha, x, 0) = g(x)h(\alpha) \quad \text{and} \quad p(\alpha, x, 1) = g(x) \tag{1}$$

The functions $g(\cdot)$ and $h(\cdot)$ will be parameterized later but notice that a worker leaving their house for work will produce an amount entirely dependent on x and output is always at least as high by working outside of the household, $p(\alpha, x, 1) \geq p(\alpha, x, 0)$. The indices α and x are drawn from a joint distribution $f(\alpha, x)$ at the time of worker-firm meeting and are fixed for the duration of the match. We allow for dependence between α and x in the distribution f and without loss of generality we assume that both have have uniform marginal distributions on $[0, 1]$.

Health status

Individuals transit between three health states h: susceptible (s), infected (i) and recovered (r) according to the standard SIR epidemiology model, with one modification. Susceptible agents who do not leave the home to work contract the disease with a Poisson rate $\lambda_0 \ell_{it}$ where $\lambda_0 > 0$ is an exogenous fixed parameter and ℓ_{it} is the share of the population who are infected at time t. We depart from the standard model by assuming that if a susceptible individual goes to work outside of their home they increase their rate of infection and become infected at an increased Poisson intensity $(\lambda_0 + \lambda_1)\ell_{it}$, where $\lambda_1 > 0$.[2] This introduces a clear trade-off for the worker, by working away from home their production will increase, and in turn so will their wage. However, they do so by increasing the likelihood of contracting the disease. Further, while a worker's decision will internalize the individual cost of working away from home it does not internalize the cost to society. By becoming infected the share of the infected population ℓ_{it} will increase and so will the rate of infection at which susceptible workers of any age and employment status catch the disease.

[2] In the model workers will not run into infected colleagues at their place of work. We think of this increased risk through travelling to work and increased exposure to other members of society while at their place of work.

Once infected, individuals will either recover from the disease at Poisson rate ρ and transition to the recovered state or they pass away from the disease at rate $\gamma_a(\ell_{it})$. We allow the mortality of the disease to vary with both an individual's age and the share of the population infected. We allow variation in age as data on recorded mortality rates differ starkly across age groups and we allow the death rate to vary with the infection rate as a proxy for capacity constraints in intensive care units.Further, in our model being infected means a worker is not able to either look form employment if out of work or produce output if in work. Finally, being recovered is an absorbing health status.[3]

The labor market

The labor market is subject to search frictions. Unemployed and healthy workers can costlessly search for a job. Firms post vacancies at flow cost κ to attract potential applicants. The total measure of vacancies posted is determined by a free entry condition. On the worker's side, only the young, non-employed and non-infected can search for work. Searching workers, $s_t := u_{st} + u_{rt}$, where u_{st} (u_{rt}) is the measure of susceptible (recovered) unemployed workers at time t, and unfilled vacancy, v_t, meet at a rate determined by a constant returns to scale meeting function $m(s_t, v_t)$. This implies a job finding rate for workers of ϕ_t and a worker finding rate of ϕ_t^f for firms,

$$\phi_t = \frac{m(s_t, v_t)}{s_t} \quad \text{and} \quad \phi_t^f = \frac{m(s_t, v_t)}{v_t} = \phi \frac{s_t}{v_t} \tag{2}$$

After meeting, the worker and firm draw α and x from the joint distribution f. There is no private information and the values of α, x and the health status of the worker will determine whether the meeting results in a match. Matches exogenously separate at a constant exogenous Poisson rate δ.

Contracting space

The joint surplus generated from a match is shared between worker and firm according to a Nash bargaining protocol. In a first step a contract is written to account for time devoted to working from home by maximizing joint surplus. Let $m \in \{0, 1\}$ be an indicator to denote if work is only performed at home, $m = 0$, or away from home, $m = 1$, respectively. Wage is determined to split the surplus according to the standard Nash sharing rule, where worker receives a share $\beta \in (0, 1)$ of the total surplus and the firm $(1 - \beta)$.

[3]At the time of writing, there is little evidence regarding antibody immunity or lack thereof. We take the stance that those who have recovered from the virus can not contract it again. Note, the model could easily accommodate a Poisson rate from recovery back to susceptibility and depending on the epidemiological evidence we may change this in future work.

We denote the value functions of matched workers and firms as $W_{ht}(w, \alpha, x, m)$ and $J_{ht}(w, \alpha, x, m)$, where W is the value of being employed and J the value of a filled vacancy for the firm in a match with a worker of health status $h \in \{s, i, r\}$, with job characteristics (α, x) under a contract (w, m) at time t. Let U_{ht} be the value of being unemployed for a worker with health status $h \in \{s, i, r\}$ and V_t be the value of an open vacancy. We assume that the joint surplus of a match can be written independent of the wage and is given by equation (3), (which is verified ex-post)

$$S_{ht}(\alpha, x, m) = W_{ht}(w, \alpha, x, m) - U_{ht} + J_{ht}(w, \alpha, x, m) - V_t \tag{3}$$

Thus when a worker and firm meet they decide jointly on the working arrangements and choose m according to

$$\arg \max_{m \in \{0,1\}} \{S_{ht}(\alpha, x, m)\} := S_{ht}(\alpha, x).$$

Since there is an ϵ cost associated with working outside the home we break the indifference by assuming if the surpluses are equal a worker works exclusively from home. We can define the set of feasible matches as the values of h, α and x that generate non-negative joint surplus, $S_{ht}(\alpha, x) \geq 0$.

Finally, after negotiating a wage and work environment both parties must comply to their contractual agreement for a stochastic length of time. We assume that if there is a change in the health status of the worker the pair can costlessly change the agreement of working at or away from home, but not their wage agreement. Otherwise they can only adjust the hours of work or wages when they re-negotiate, which happens at an exogenous Poisson rate ν. After the re-negotiation shock they may also decide to separate if the joint surplus is negative. This rigidity models in a reduced form way the inability of UK firms to layoff workers immediately after changes in policy or worker's changing health status.

Vacancy creation

Vacant jobs make contact with unemployed workers at a rate ϕ_t^f. We assume free entry such that potential firms continue to post vacancies until the presented discounted expected value of doing so is zero. The value of posting a vacancy is given by

$$\begin{aligned} rV_t = -\kappa + \phi_t^f (1-\beta) \Bigg(& \frac{u_{st}}{u_{st} + u_{rt}} \int \int \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\} dF^2(\alpha, x) \\ & + \frac{u_{rt}}{u_{st} + u_{rt}} \int \int \max\{S_{rt}(\alpha, x), 0\} dF^2(\alpha, x) \Bigg) \end{aligned} \tag{4}$$

where κ is the flow cost incurred when posting a vacancy. Thus the equilibrium aggregate number of vacancies are determined by setting the left hand side of equation (4) to zero.

Equilibrium and solving the model

The model structure allows all decisions, whether a worker-firm match is feasible and if so whether the worker should work in or away from the household, to be a function of the joint surplus of a match. This property is shown by specifying and solving the value functions in Appendix A.1. In addition one must compute the allocation of workers across demographic, health and economic status. These follow the dynamics in Appendix A.2. We assume the economy starts from a unique steady-state in which the whole population is susceptible and deviate with a *small* initial seed mass in which the probability of infection is constant across employment state. The final equilibrium object to pin down is the number of vacancies posted by firms, which given worker allocations and surpluses uniquely solves equation (4). Details of how these objects are computationally solved are provided in Appendix A.3.

3 Economy under lockdown

Lockdown is modeled as an exogenous and random share $\pi \in [0, 1]$ of the economy prevented from operating away from home (e.g. an office). Workers in these locked jobs are mandated to only work at home. Thus if the policy binds, a match of production index α will see their production fall by a share $(1 - h(\alpha))$. New jobs can either be in the '*locked*', (with probability π), or '*unlocked*' sector, (with probability $(1 - \pi)$). This draw is made at the time of worker-firm meeting and is assumed orthogonal to α and x.[4]

We model lockdown as slowing the rate of transmission through two mechanisms. Firstly, fewer people work away from their home. This reduces the number of people who contract the disease at their place of work. Those working at home have a Poisson rate of becoming infected which is $\lambda_1 \ell_{it}$ less than those working away from home. The second mechanism is through social distancing. While not explicitly modeled, a lockdown on bars and restaurants for example will reduce the number of social interactions in the economy. The parameter λ_0 governs the latent transmission rate irrespective of working decisions. Since lockdown will also reduce this we assume this parameter in lockdown is given by

$$\lambda_0^L := (1 - \zeta\pi)\lambda_0.$$

$\zeta \in [0, 1]$ governs the potential effectiveness of the lockdown. For example, if the government took the extreme action of shutting down the entire economy the Poisson rate of

[4]In future work, when survey data can be easily analyzed it would be interesting to assume two conditional distributions for $f(\cdot)$. This would allow one to evaluate economic costs from targeted lockdowns.

infection for a susceptible individual would be given by $(1-\zeta)\lambda_0\ell_{it}$. The final amendment to the model is that lockdown is not permanent. While lockdown arrives as an unanticipated shock, agents assume it ends at an exogenous Poisson rate Λ after which the economy returns to the status quo. Modeling lockdown policy introduces an additional state variable for a worker-firm pair. That is, whether or not the job is '*locked*' or *unlocked*', otherwise the model retains the same structure. In order to avoid repetition, we relegate the exposition and solution of the model to a complementary online appendix.

4 Quantitative results

The goal of this section is to examine the likely effects of lockdown policy on the safety of workers and the performance of the economy as a whole. Rather than being explicit about a social welfare function we simply demonstrate the tradeoff between the likely number of fatalities from the epidemic and the stress to the economy caused by lockdown policy. It is necessary to begin with two home truths. Firstly, a laissez faire approach, in the presence of the pandemic, will cause an economic downturn. That is to say, because of endogenous responses in the model, even in the absence of economic policy there will be economic losses and they are likely to be large. In particular, we find cumulative output losses to be around 2.4% of the pre-pandemic level under the laissez faire approach over 5-year horizon. Secondly, in the absence of a vaccine, the infection exists indefinitely, irrespective of how draconian a lockdown policy may be. In fact because we model new entrants into the labor market as susceptible, in the long run the pandemic will repeat itself in dampening cycles in perpetuity. Since these cycles materialize at approximately a twenty year frequency, we abstract from these in our discussion of policy and assume by the time of the next cycle a vaccine has been developed. Consequently, all discussion will relate to the ongoing wave of the pandemic. As a preview of our results we summarize these points and other findings in the list below.

1. Lockdown will not rid us of the virus. For that a vaccine needs to be found.

2. Lockdown is not the only source of economic stress. The economy will suffer from a laissez-faire approach.

3. Lockdown policy can mitigate the loss of life in this wave but to be effective in saving lives it will have to be implemented for a considerable length of time.

4. The economic costs of lockdown are not borne uniformly across the cross-section of workers: those at the lower-end of the wage distribution are affected disproportionally more.

5. Given the response of the labor market to different lockdown measures and the heterogeneity in economic costs — we make a policy prescription to be run in conjunction with lockdown to mitigate economic losses.

4.1 Parameterization

To proceed, we begin by specifying functional forms for the matching function $m(s_t, v_t)$, the functions entering production, $g(x)$ and $h(\alpha)$, the distribution of job's characteristics, $f(x, \alpha)$, and the death rates for young and old individuals, $\gamma_o(\ell_{it})$ and $\gamma_y(\ell_{it})$. We use a standard Cobb-Douglas function to model contacts between vacancies v and searchers s

$$m(s_t, v_t) = s_t^{1-\xi} v_t^{\xi},$$

where $\xi \in (0, 1)$ denotes the elasticity of contact with respect to the stock of open vacancies. This matching function implies

$$\phi_t^f = \frac{m(s_t, v_t)}{v_t} = \theta_t^{\xi-1} \quad \text{and} \quad \phi_t = \frac{m(s_t, v_t)}{s_t} = \theta_t \phi_t^f.$$

The variable θ_t denotes the labor market tightness, defined as $\theta_t := v_t/s_t$. We specify the total potential output of a worker-firm pair of index x as the inverse of a log-normal distribution with underlying mean μ_x and variance σ_x^2,

$$g(x) = \exp\left(\mu_x + \sigma_x \Phi^{-1}(x)\right)$$

where $\Phi^{-1}(\cdot)$ denotes the inverse cumulative distribution function of a standard normal distribution. To describe the proportion of job tasks that can be performed at home, we assume $h(\alpha)$ to be an inverted Beta distribution,

$$h(\alpha) = \frac{\alpha^{\beta_1 - 1}(\alpha + 1)^{-\beta_1 - \beta_2}}{\mathcal{B}(\beta_1, \beta_2)}$$

where $\beta_1, \beta_2 \geq 1$ are the parameters of the beta distribution and $\mathcal{B}$ denotes the Beta function. To model correlation between job productivity x and home efficiency α, we choose the function $f(\alpha, x)$ to be a Gaussian copula with correlation parameter $\rho_{\alpha,x}$. Finally, we assume the death rates for old and young individuals are independent of the stock of infected, and we set them equal to

$$\gamma_o(\ell_{it}) = \gamma_o \quad \text{and} \quad \gamma_y(\ell_{it}) = \gamma_y.$$

4.2 Calibration

The model is calibrated at a weekly frequency for the pre-epidemic period and simulations are run at a daily frequency. The first three blocks of Table 1 report parameters values for demographic, labor market and technology and the moments used to calibrate them. The interest rate r is set to have an annual return of 5%. Workers spend on average 40 years in the labor market, and 15 years in retirement. These values pin down ageing rate η and death rate χ.

We set the re-negotiation rate to match two weeks of advance notice and fix $\nu = 0.5$.[5] We set the income flow for unemployed workers to 65% of the average wage as reported for the UK in 2019 by the OECD. The income flow for retired workers to 75% of the average wage, to match the ratio between equivalized disposable income of retired and non-retired HH (ONS). The bargaining power, β, is calibrated to match a value for labor share equal to 54.63% (UK national accounts 2016Q3). The matching elasticity, ξ is calibrated to match the estimated value of 0.35 in Turrell et al. (2018). The exogenous job destruction rate, δ, is calibrated to match a monthly separation rate of 4% reported in Postel-Vinay and Sepahsalari (2019).[6] Finally, we calibrate the cost of posting of vacancy, κ, to match the employment rate in the last quarter of 2019 (ONS).

We are left with five parameters, governing productivity and home-working efficiency. We calibrate the parameters in the output technology μ_x and σ_x to match an average weekly earnings of 545 GBP (ONS Weekly Earnings Survey, February 2020) and an average stock of vacancy per population in the last quarter of 2019 of 1.19% (ONS - Vacancy Survey).[7] Conditional on all other parameters, including the meeting function, the number of matches from a stock of vacancies is governed by the proportion of meetings that result in matches. This proportion is driven by the degree of dispersion in the job sampling distribution.[8] While we choose the parameters in the inverted beta distribution, β_1 and β_2 to match average and standard deviation of home-working hours before the pandemic. These data are taken from January-December 2019 from the Annual Population Survey (APS) and presented in Appendix A.4, panel a. Finally, we calibrate the copula parameter, $\rho_{\alpha,x}$, to match the correlation between number of hours and average log weekly earnings (see Appendix A.4, panel b)

Turning to the parameters of the SIR model, we follow Ferguson et al. (2020) and calibrate

[5] The statutory redundancy notice period in the UK is in practice a function of the length of time one has been in their job. Those employed for under a month can be laid off without notice. For those employed between one month and two years, one week notice is required. Then for each additional year a further weeks notice is required, capped at twelve weeks.

[6] Recall, we do not distinguish between the young and inactive and unemployed so take the sum of the separation rates to unemployment and inactivity at the end of their sample.

[7] Expressions for wages in our model are deferred to the online appendix.

[8] To see this, imagine there were no dispersion in productivity. All worker-firm meetings will result in matches as the worker or firm have no incentive to wait and find a better match.

Table 1: Calibrated Parameters

Parameters	Description	Value	Source/Target
			Demographics
r	Discount rate	0.00098641	Annual return: 5% annual
η	Ageing rate	0.00048077	40 years in the labor market: 25-65 y.o.
χ	Death rate	0.00128210	15 years of retirement: 65-80 y.o.
ψ	Birth rate	0.00034965	Pre-epidemic population=1
			Labor market
ν	Re-negotiation rate	0.5	Two weeks advance notice
b_r	Retirement income flow	406.02	Equivalized disposable income retired/non-retired HH=75% (ONS)
b_u	Unemployment income flow	354.25	Average replacement rate=65% (OECD)
ξ	Matching elasticity	0.35	Turrell et al. (2018)
β	Bargaining power	0.0988	Labor share=54.63% (ONS)
δ	Job destruction rate	0.010205	Monthly job separation=4% Postel-Vinay and Sepahsalari (2019)
κ	Vacancy cost	60763	Employment rate=76% (ONS)
			Technology
β_1	Home-working efficiency	0.0510	Average home-working hours: E[h]= 11.55% (APS)
β_2	Home-working efficiency	3.3780	St.Dev. home-working hours: std[h]= 9.99% (APS)
μ_x	Output technology	4.1997	Average weekly earnings: E[w]= 545 (ONS)
σ_x	Output technology	1.5252	Vacancy per population= 1.19% (ONS)
$\rho_{\alpha,x}$	Copula parameter	0.9578	Corr. log weekly earnings and home-working hours: corr[logw, h]= 0.713 (APS)
			Epidemic dynamics
λ_0	Infection rate, basic	1.6759	Basic reproduction rate: $R_0 = 2.4$, Ferguson et al. (2020)
λ_1	Infection rate, at work	0.0728	Infection at work: 0.024, Riccardo et al. (2020)
γ_y	Death rate, young	0.00225	Case fatality ratio: death/cases 0.32% Verity et al. (2020)
γ_o	Death rate, old	0.04795	Case fatality ratio: death/cases 6.4% Verity et al. (2020)
ρ_y	Recovery rate, young	0.7	Average recovery period: ten days, Ferguson et al. (2020)
ρ_o	Recovery rate, old	0.7	Average recovery period: ten days, Ferguson et al. (2020)
			Practicalities
	Initial seed mass	10^{-9}	
	First death	$1/66/10^6$	
	Burnin period	24 days	Time between first death and lockdown in the UK.
			Lockdown
π	Share of economy on lockdown	0.63	Change in visits to '*workplace*', Google location data
ζ	Social distancing parameter	0.67	Change in visits to '*retail and recreation*', Google location data

λ_0 and λ_1 to match an average basic reproduction rate of 2.4 at the eve of the epidemic breakthrough. From the context of the model this is the reproduction rate when the entire population is susceptible without any endogenous changes to the working environment. From the perspective of the data, this comes from the early estimates in Wuhan, again when the population was close to fully susceptible.[9] To disentangle the value of λ_0 from λ_1 we calibrate

[9] Estimates from Riou and Althaus (2020) and Li et al. (2020) put the number somewhere between 2.0 and 2.6

λ_1 to match the number of individuals who contracted the virus at their place of work. Riccardo et al. (2020) estimate this number in Italy as being at 2.4%. We calibrate death rates of young and old, γ_y and γ_o to match case fatality ratio in their age categories Verity et al. (2020)., Finally, we fix the average recovery period to 10 days following Ferguson et al. (2020).

4.3 Counterfactual experiments

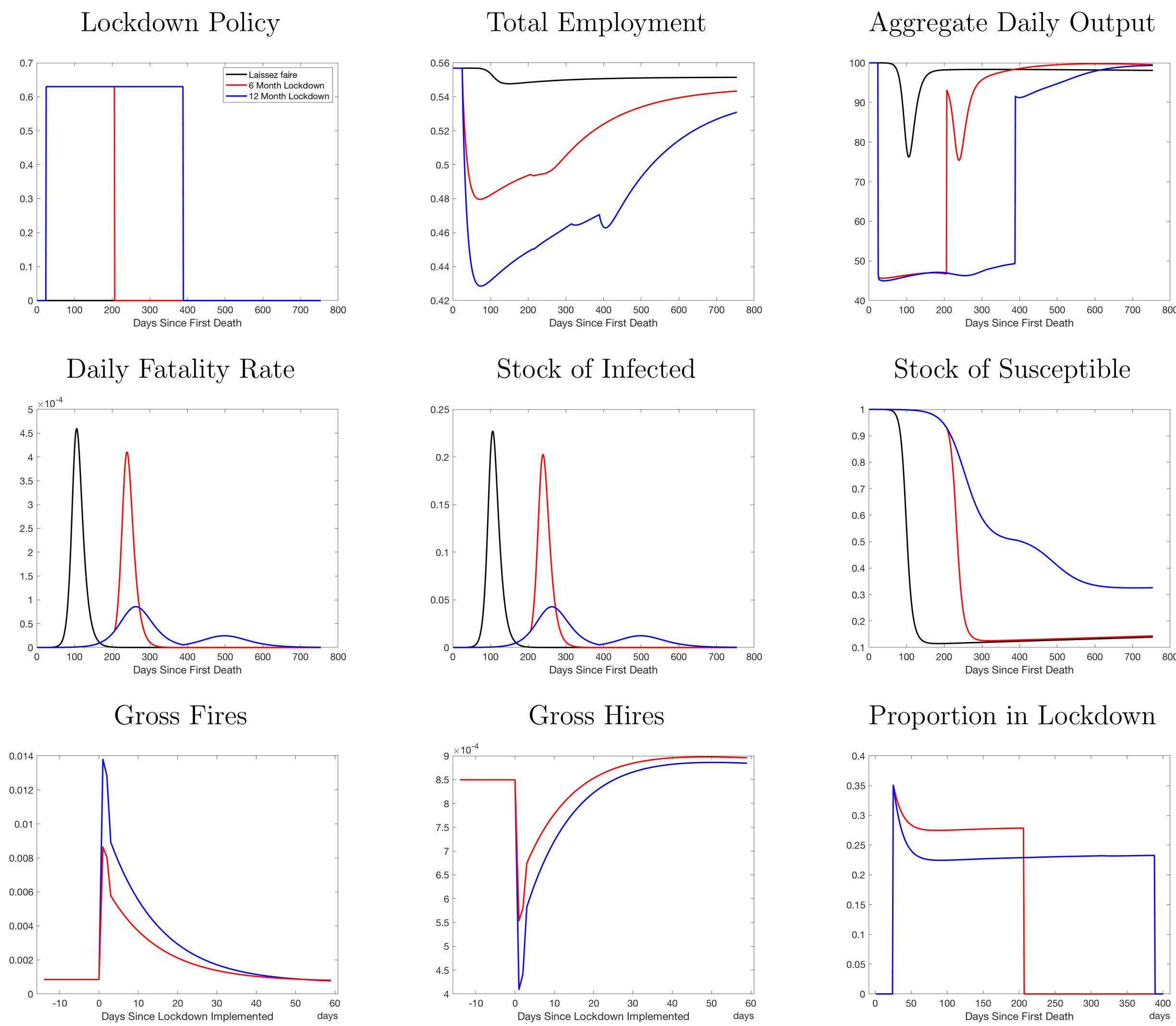
We keep the severity of a lockdown (π) fixed and vary the duration (Λ). The specifics of the policy simulation are represented in the final block of Table 1. We begin with very few infected people and assume all employment states are equally likely to be infected at time zero. The economy is simulated and we assume lockdown arrives as an unanticipated shock 24 days after the first registered death, to mirror the experience of the UK. Since there are a continuum of workers in the model we interpret the first death as the number of deaths exceeding one divided by the UK's population. The proportion of the economy locked down π and the impact this has on social distancing ζ are calibrated from Google user's location data. The proportion of jobs locked down is taken from the change in visits to the user's workplace which dropped by 63% post lockdown. The impact this had on social distancing is the taken from the change in visits to a '*social*' sector that was largely locked down, retail and recreation. This includes retail outlets, shopping centers, museums etc. but omits grocery stores and other more essential services that were not placed on lockdown.

Health costs. We begin by looking at the health costs of the pandemic associated with a six and twelve month lockdown period. The lockdown policy is shown in the first panel of Figure 2 and the associated health costs in the second row. Both lockdown policies are able to suppress the pandemic to some extent and will result in fewer total deaths than doing nothing shown in black. The six month lockdown suppresses the virus during lockdown but is lifted before the peak and results in many more lives lost following the lifting of restrictions. By contrast the twelve month lockdown appears to break the back of the pandemic. However, because of the tightness of the restrictions there are still many susceptible individuals in the economy, below the level needed for herd immunity. Hence after the lifting of the restrictions a second wave of the virus sweeps through the population.[10]

Economic cost. As well as variation in the health costs associated with different lockdown policies there are also large variations in the economic consequences. As has been

[10] A less strict lockdown policy as measured by π could actually lower the fatality rates in this illustrative example. For the case of the six month policy, the peak would come sooner and could potentially be dampened with more early exposure. Similarly, if calibrated perfectly, the twelve month lockdown with a lesser π would have a larger initial wave but could avoid the second wave by reducing the number of susceptible people still in the economy.

Figure 2: Dynamics of the Pandemic and Economy



discussed no policy intervention is not costless from an economic point of view. Work days are lost because of illness and the increased exposure to health risks reduce the value of jobs and thus the level of vacancy posting reduces. Lockdown policy will inevitably confound these losses. Primarily because it directly reduces potential output, forcing a share of jobs in the economy to limit production to inside the worker's home. Clearly, the longer the economy is restricted, the larger these losses are going to be. However the losses are also intrinsically linked to the workings of the labor market. This can be seen in the first row of Figure 2. The shorter lockdown has a much smaller initial fall in employment. Since firms know the lockdown is relatively short, firms opt to hoard their workforce. Even in the face of a considerable drop in production, firms prefer this choice over incurring hiring costs in the future; they keep their workers on the payroll and take the short term losses. This labor hoarding has advantages and disadvantages to the economy. On the positive side, it makes for a speedier recovery when lockdown ends. As can be seen in the figure, daily output returns to pre-crisis levels quicker for the shorter lockdown. Since many more matches are held together through the pandemic they are well suited to the *normal* environment post lockdown. However, during the lockdown production is likely to stay low as the labor market does not readjust to the changing environment.

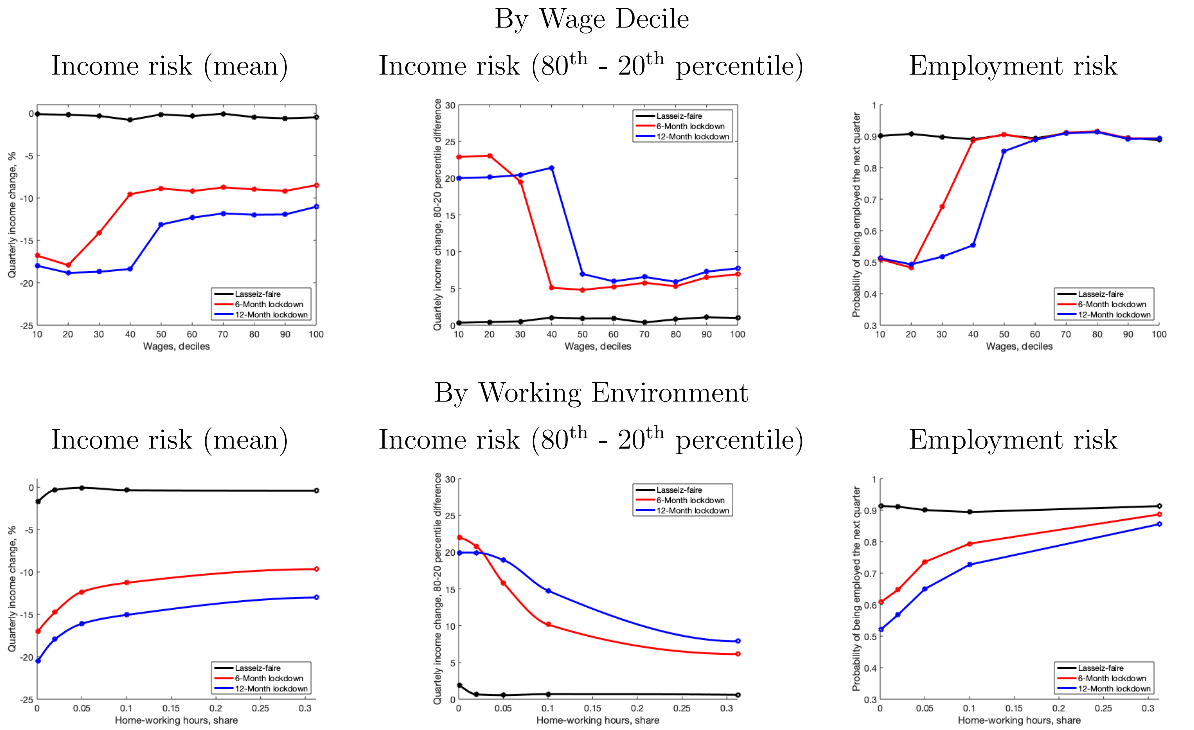
Labor adjustment. To better understand the different labor market response to the different durations of lockdown, the final row of Figure 2 plots the response in gross hiring and firing following implementation. As discussed, the more severe lockdown results in many more layoffs as hoarding labor for prosperous times to come becomes far more expensive. At the same time there is also a large initial fall in hiring as many matches are locked and will not hire unless they are extremely productive or efficient in working from home. After an initial fall, the level of hiring rises steadily under both regimes. This is in part due to a larger pool of unemployed following the large rise in layoffs and in part because of workers' falling outside option — the deteriorating state of the economy makes them less discerning in which matches to accept. In fact, because of the enormous misallocation shock to the economy, hiring levels under both policy options eventually exceed the level of hiring pre-lockdown. The final panel shows the direction that reallocation takes. Initially the share of workers subject to lockdown is the same as the proportion of the economy under lockdown. However, following layoffs based predominantly in locked sectors, in addition to new hires going into matches that are unlocked, there is a decline in the fraction of the economy locked down.

Heterogeneous effects. Figure 3 depicts the effect of the short and long duration lockdown policies, in addition to a laissez-faire approach, on measures of income and employment risk for a simulated panel of workers. To study the effect on income, we look at total income of a worker in the first quarter of lockdown relative to the last prior to its commencement. Our measure of employment risk is the probability that a worker, who is employed at the time of lockdown's commencement, remains employed in the next quarter.

Quarterly income is computed as the integral of all earnings over a quarter, both labor, and if unemployed, home production. For a susceptible worker, who form almost the entire population at the implementation of lockdown, see Figure 2, the surplus of a match will fall. This is true irrespective of lockdown status as there is an increased probability of match disruption, through the worker getting infected. For those in locked professions this fall in surplus and hence wages is confounded further as output will fall considerably.[11] In unlocked

[11]For a worker in a match (α, x) who if unlocked would leave the home for work will see a proportional fall in output of $(1 - h(\alpha))$. On average that corresponds to an almost 90% fall in output.

Figure 3: Heterogeneous effects of policies on the worker cross-section

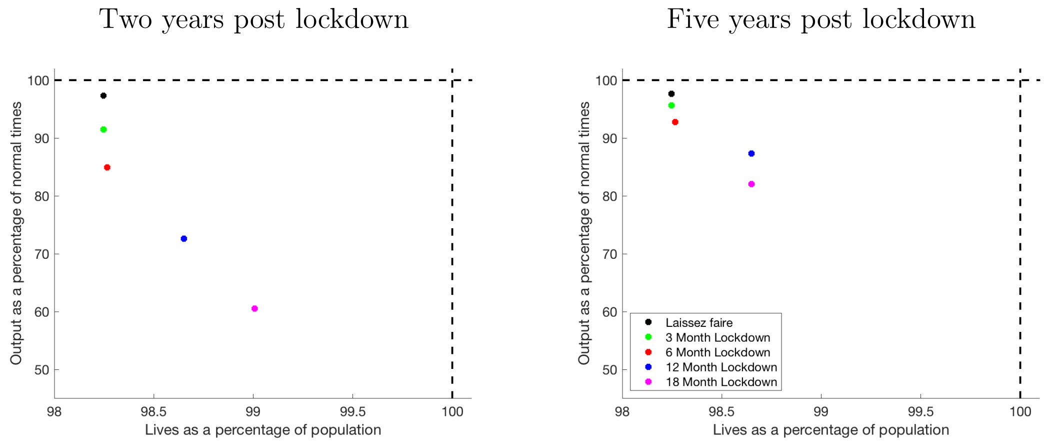


Notes: 20,000 workers are simulated over two quarters either side of the implementation of lockdown. Included in the sample presented in the figure are those who were employed at lockdown and were active in the labor market (neither retired nor newly entered) for the 90 days prior and subsequent to lockdown. Leaving a reduced sample of 10,864.

professions the change in wage is ambiguous and will vary from match to match. On the one hand, the surplus falls through increased disruption. On the other, to compensate the increased risk exposure the worker will take a larger share of the output. In addition to wage changes on the job the other source of income risk are endogenous separations. If the surplus falls below zero a worker and firm match separates. A terse glance at Figure 3 reveals this second mechanism is the primary driver in increased income risk.

Comparing mean income falls with employment probabilities by either wage or work environment show the same workers are losing out in both. Inspection of Figure 3 reveals the aggregate implications of these different mechanisms for workers across the distribution of wages and working environment. The first column shows the average change in quarterly income. While lockdown policy is ubiquitous in its impact those that spend little time working at home or are low wage workers suffer a lot more. As discussed, looking at the third column it is easy to see why. These are the workers who are being laid off and consequently

Figure 4: Policy Possibility Frontier



suffer large losses in income. For sufficiently well paid workers there is no increase in employment risk with lockdown. This cutoff increases as the severity of the lockdown increases. Finally, in addition to increased employment risk and lower income, the risk (measured as the dispersion of income changes) also increases. This is felt hardest by those with the least ability to insure against it, the lowest paid.

Evaluating policy options. Rather than being explicit about a social welfare function we follow Kaplan et al. (2020) and define a policy possibility frontier. This function is useful for policymakers as it plots the feasible outcomes, lives saved and economic consequences of different lockdown policies. Taking two and five year horizons, Figure 4 plots differing durations of lockdown on this health-economic space. One can see the clear tradeoff between the two metrics; a judgment on how draconian a policy a government wishes to implement will depend on its specific welfare function. Given insights from the quantitative model we instead discuss how to give policymakers a better menu of outcomes. That is: what labor market policy used in conjunction with lockdown could shift the frontier in a north easterly direction. In particular we consider the 'Coronavirus Job Retention Scheme' implemented by the UK government.

The key findings that drive our policy discussion are the following. First, during a short lockdown, there is a large degree of labor hoarding, which speeds-up the economic recovery when the restrictions are lifted. Second, this labor hoarding suppresses economic activity during the lockdown. Third, young workers on a low wage are those harmed the most economically from the lockdown. These three results speak directly to the efficacy of the furlough aspect of the UK Government's Scheme.

The furlough scheme implemented paid workers 80% of their monthly salaries, capped at 2,500 pounds per month. The policy eventually allowed workers to seek alternative employment while furloughed and expanded to the self-employed as well. The impulse response functions in Figure 2 illustrate why this policy would mitigate economic costs associated with a lockdown. Allowing worker-firm pairs to temporarily separate without breaking their employment ties will foster a quick recovery following the end of lockdown, as with the short lockdown exercise. Crucially however allowing workers to seek alternative employment in the short run allows for greater production during lockdown — either in the unlocked sectors, which are likely vital, or in locked sectors that do not rely so much on working outside of the home.

A similar conceptual point is made by Fujita et al. (2020) and Costa-Dias et al. (2020) whom lobbied the government to switch track and allow furloughed workers to seek other alternative temporary employment. Finally, a large component of the negative income risk brought on by the pandemic was employment risk and this is particularly felt by low wage workers. By taking the wage burden away from the firm, the government can insure against this risk. If policymaker also cared about inequality this would further improve outcomes. This mechanism is discussed by Blundell et al. (2020). In this paper we regard our policy prescription as a proof of concept. Future quantitative work to get a handle on just how useful such a policy could be extremely fruitful. To do this one would have to explicitly model a labor search model with *job memory* with an epidemiology model.[12]

5 Conclusion

This paper combines two workhorse models from labor economics and epidemiology to create a choice theoretic model of disease transmission and a frictional labor market. Worker-firm decisions about whether to work from home and firm's vacancy decisions are consequential for the state of the economy and crucial for the infection rate. Understanding the co-movement of the pandemic and labor market is crucial for policymakers especially when deciding on lockdown policies. We show that the response of both differ starkly given the length of the lockdown imposed. Finally, we use insights garnered from the quantitative model to support the UK government's 'Coronavirus Job Retention Scheme'.

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A Appendix

A.1 Surplus functions of baseline model

The demography of the model has workers moving from working age to retiring to death and the health dynamics from susceptible, to infected, to recovered, conditional on survival. We present the value functions in the same order the model is solved. Starting with terminal conditions and working backwards.

Retired workers

We begin with a retired individual who has recovered from the illness. The index t encapsulates all potential aggregate state variables that vary with time. The discounted value is the sum of the flow value worker's get after retiring b_o and the option value of death, which occurs at Poisson rate χ.

$$rR_{rt} = b_o + \chi(0 - R_{rt}) + \dot{R}_{rt}$$

It can be seen that this value function is independent of time and can be rewritten dropping the time subscript as

$$R_r = \frac{b_o}{r+\chi}.$$

Retired agents who are currently infected have an increased death probability of $\gamma_o(\ell_{it})$ which varies with time through the evolution of the proportion of sick people. Additionally, they can recover from their illness at a rate ρ_o.

$$\begin{aligned} rR_{it} =& b_o + (\chi + \gamma_o(\ell_{it}))(0 - R_{it}) + \rho_o(R_r - R_{it}) + \dot{R}_{it} \\ (r + \chi + \gamma_o(\ell_{it}) + \rho)R_{it} =& \frac{r+\chi+\rho_o}{(r+\chi)} b_r + \dot{R}_{it} \end{aligned}$$

Finally, retired agents who are susceptible again die at the reduced rate χ but they can also become infected which again depends on the proportion of the population with the infection at time t.

$$\begin{aligned} rR_{st} =& b_o + \chi(0 - R_{st}) + \lambda_o\ell_{it}(R_{it} - R_{st}) + \dot{R}_{st} \\ (r + \chi + \lambda_o\ell_{it})R_{st} =& b_o + \lambda_o\ell_{it}R_{it} + \dot{R}_{st} \end{aligned}$$

Recovered young individuals

The value of being unemployed for a recovered individual is the sum of four terms. (i) The flow benefit b_u they get from being out of work. This encapsulates both pecuniary and non-pecuniary benefits including for example the value of leisure time. (ii) The option value of finding a job, from which if the surplus is positive they will get a fraction β of. Offers arrive at an endogenous rate ϕ_t to be determined later. (iii) The option value associated with retirement which occurs at exogenous rate η. (iv) The continuation value from dynamic changes to the offer arrival rate and infection rate. These four terms are represented in the Bellman equation below.

$$rU_{rt} = b_u + \phi_t\beta \int\int \max\{S_{rt}(\alpha, x, 1), S_{rt}(\alpha, x, 0), 0\}d^2F(\alpha, x) + \eta(R_{rt} - U_{rt}) + \dot{U}_{rt}$$

The value of being employed in a job of match (α, x) for a recovered individual in a contract (w, m) is given below. Where $w \in \mathbb{R}$ is the contractually agreed wage and $m \in \{0, 1\}$, taking the value one if the worker leaves their abode to work and zero otherwise.

$$\begin{aligned} rW_{rt}(w, \alpha, x, m) &= w + \delta(U_{rt} - W_{rt}(w, \alpha, x, m)) + \eta(R_{rt} - W_{rt}(w, \alpha, x, m)) \\ &\quad + \nu\left(\max\{\beta S_{rt}(\alpha, x, 1), \beta S_{rt}(\alpha, x, 0), 0\} + U_{rt} - W_{rt}(w, \alpha, x, m)\right) + \dot{W}_{rt}(w, \alpha, x, m) \end{aligned}$$

Value of filled vacancy. The value of an employer in a match (α, x) with a recovered individual and contract (w, m) is equal to

$$\begin{aligned} rJ_{rt}(w, \alpha, x, m) &= p(\alpha, x, m) - w + (\delta + \eta)(V_t - J_{rt}(w, \alpha, x, m)) \\ &\quad + \nu\left((1 - \beta)\max\{S_{rt}(\alpha, x, 1), S_{rt}(\alpha, x, 0), 0\} + V_t - J_{rt}(w, \alpha, x, m)\right) + \dot{J}_{rt}(w, \alpha, x, m). \end{aligned}$$

The flow value the firm receives is the production of the match, which will depends on whether the worker leaves their home ($m = 1$) or not ($m = 0$), net of the worker's wage w. From the firm's perspective whether a worker leaves to unemployment or to retirement is immaterial to them. Otherwise the option values are as in the case of the employed worker.

Value of surplus. Imposing free entry, $V_t = 0$, the surplus value for a match (α, x) in a contract (w, m) is derived by substituting the above expressions into equation (3).

$$\begin{aligned} (r + \delta + \eta)S_{rt}(\alpha, x, m) &= p(\alpha, x, m) - b_u - \phi_t\beta \int\int \max\{S_{rt}(\alpha, x, 1), S_{rt}(\alpha, x, 0), 0\}d^2F(\alpha, x) \\ &\quad + \nu\left(\max\{S_{rt}(\alpha, x, 1), S_{rt}(\alpha, x, 0), 0\} - S_{rt}(\alpha, x, m)\right) + \dot{S}_{rt}(\alpha, x, m) \end{aligned}$$

Since $p(\alpha, x, 1) \geq p(\alpha, x, 0)$, it is easy to show that $S_{rt}(\alpha, x, 1) \geq S_{rt}(\alpha, x, 0)$. In fact:

$$S_{rt}(\alpha, x, 1) - S_{rt}(\alpha, x, 0) = \frac{(1 - h(\alpha))g(x)}{r + \delta + \eta + \nu} \geq 0.$$

Therefore for brevity of notation we set $S_{rt}(\alpha,x)=S_{rt}(\alpha,x,1)$, $J_{rt}(w,\alpha,x)=J_{rt}(w,\alpha,x,1)$ and $W_{rt}(w,\alpha,x)=W_{rt}(w,\alpha,x,1)$.

Infected young individuals

Infected unemployed are too ill to search for a job. Their value functions is equal to:

$$rU_{it}=b_u+\rho_y\left(U_{rt}-U_{it}\right)+\gamma_{0y}(\ell_{it})(0-U_{it})+\eta(R_{it}-U_{it})+\dot{U}_{it}.$$

In addition to the flow value associated with any unemployment their option values consist of recovering and becoming unemployed and recovered, passing away in which case they get nothing, and retiring. Infected individuals are too ill to work, but receive a sick pay w, and they return to their job upon recovery. The value for the employed infected is equal to

$$\begin{aligned}rW_{it}(w,\alpha,x)=w&+\rho_y\left(W_{rt}(w,\alpha,x)-W_{it}(w,\alpha,x)\right)\\&+\gamma_{0y}(\ell_{it})(0-W_{it}(w,\alpha,x))+\delta\left(U_{it}-W_{it}(w,\alpha,x)\right)+\eta\left(R_{it}-W_{it}(w,\alpha,x)\right)\\&+\nu\left(\beta\max\{S_{it}(\alpha,x),0\}+U_{it}-W_{it}(w,\alpha,x)\right)+\dot{W}_{it}(w,\alpha,x)\end{aligned}$$

Other than sick pay, the value of employed infected accounts for the option value of recovering and go back to work, of passing away because of the infection, of exogenously separating, in which case they become unemployed infected, of retiring, and of renegotiating the terms of the contract, which can lead to match destruction.

Value of filled job. Employers in a match with infected employee produce nothing and are forced to a mandatory sick payment w to the worker. Their value is equal to:

$$\begin{aligned}rJ_{it}(w,\alpha,x)=-w+\rho_y\left(J_{rt}(w,\alpha,x)-J_{it}(w,\alpha,x)\right)+(\gamma_{0y}(\ell_{it})+\delta+\eta)(V_t-J_{it}(w,\alpha,x))\\+\nu\left((1-\beta)\max\{S_{it}(\alpha,x),0\}+V_t-J_{it}(w,\alpha,x)\right)+\dot{J}_{it}(w,\alpha,x)\end{aligned}$$

Employers have to option of renegotiating the terms of the contract at rate ν, which could lead to match destruction. A match can also destroy because of exogenous separation, occurring at rate δ, or because of employee death, which occurs at a rate $\gamma_{0y}(\ell_{it})$. The match starts producing again upon worker recovery, occurring at rate ρ_y.

Value of surplus. Given free entry, $V_t=0$, the surplus of a match between an employed and a sick employee can be written as follows:

$$(r+\delta+\eta+\rho_y+\nu+\gamma_{0y}(\ell_{it}))\,S_{it}(\alpha,x)=-b_u+\rho_y S_{rt}(\alpha,x)+\nu\max\{S_{it}(\alpha,x),0\}+\dot{S}_{it}(\alpha,x)$$

Notice that - even when the employee is infected - the match surplus could be positive, as long as the continuation value is larger than the unemployment flow. In this case, the match won't cease to exist, the employer will transfer a sick pay to the employee and wait till her recovery.

Susceptible young individuals

Susceptible individuals face risk of infection. The infection rate is function of the share of infected people in the economy, ℓ_{it}, and it depends on the employment status: it is equal to $\lambda_{0y}\ell_{it}$ for unemployed workers. Susceptible unemployed have the following value:

$$(r + \lambda_{0y}\ell_{it} + \eta)U_{st} = b_u + \phi_t\beta \int\int \max\{S_{st}(\alpha, x, 1), S_{st}(\alpha, x, 0), 0\}d^2F(\alpha, x) + \lambda_{0y}\ell_{it}U_{it} + \eta R_{st} + \dot{U}_{st}$$

which depends on the unemployment flow plus the option value of finding a jobs, getting infected unemployed, and retiring as susceptible. Susceptible employed differ by their job characteristics (α, x) and their contractual arrangements, (w, m), which in turns determine their rate of contagion. Employees working only from home ($m = 0$) get infected at the same rate of unemployed workers while employees working away from home get infected at a larger rate, equal to $(\lambda_{0y} + \lambda_{1y})\,\ell_{it}$, where λ_{1y} governs the rate of contagion at work. The value of employment for susceptible workers reflects these differences and it is equal to:

$$(r + \delta + \nu + \lambda_{0y}\ell_{it} + \eta)W_{st}(w, \alpha, x, 0) = w + (\delta + \nu)U_{st} + \lambda_{0y}\ell_{it}W_{it}(w, \alpha, x) + \eta R_{st} + \nu\beta \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\} + \dot{W}_{st}(w, \alpha, x, 0)$$

if $m = 0$, and equal to:

$$(r + \delta + \nu + (\lambda_{0y} + \lambda_{1y})\ell_{it} + \eta)W_{st}(w, \alpha, x, 1) = w + (\delta + \nu)U_{st} + (\lambda_{0y} + \lambda_{1y})\ell_{it}W_{it}(w, \alpha, x) + \eta R_{st} + \nu\beta \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\} + \dot{W}_{st}(w, \alpha, x, 1)$$

if $m = 1$. Except for the infection rates, employees with different home-working arrangement have similar value of employment: their matches are exogenously destroyed at a rate δ, they retire at a rate η and renegotiate their contract a rate ν.

Value of filled job. An employer (α, x) matched with a susceptible employee produces $p(\alpha, x, 0)$ if the employee works only from home or $p(\alpha, x, 1)$ if the employees works away from home. Imposing free entry, $V_t = 0$, the value of an employer matched with a susceptible employee is equal to:

$$(r + \delta + \eta + \lambda_{0y}\ell_{it} + \nu)J_{st}(w, \alpha, x, 0) = p(\alpha, x, 0) - w + \lambda_{0y}\ell_{it}J_{it}(w, \alpha, x) + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\} + \dot{J}_{st}(w, \alpha, x, 0)$$

if $m = 0$, and equal to:

$$(r + \delta + \eta + (\lambda_{0y} + \lambda_{1y})\,\ell_{it} + \nu)J_{st}(w, \alpha, x, 1) = p(\alpha, x, 1) - w + (\lambda_{0y} + \lambda_{1y})\,\ell_{it}J_{it}(w, \alpha, x) + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\} + \dot{J}_{st}(w, \alpha, x, 1)$$

if $m = 1$. Except for exogenous match destruction or worker retirement, the option values are as in the case of the susceptible employed.

Value of surplus. Given free entry $V_t = 0$, total surplus for a match in a contract (w, m) can de defined as follows:

$$\begin{aligned}(r+\delta+\eta+\nu+\lambda_{0y}\ell_{it})\,S_{st}(\alpha,x,0) &= p(\alpha,x,0)-b_u \\ &- \phi_t\beta\int\int \max\{S_{st}(\alpha,x,1),S_{st}(\alpha,x,0),0\}d^2F(\alpha,x) \\ &+ \lambda_{0y}\ell_{it}S_{it}(\alpha,x)+\nu\max\{S_{st}(\alpha,x,0),S_{st}(\alpha,x,1),0\} \\ &+ \dot{S}_{st}(\alpha,x,0)\end{aligned}$$

if $m = 0$, and equal to

$$\begin{aligned}(r+\delta+\eta+\nu+(\lambda_{0y}+\lambda_{1y})\ell_{it})S_{st}(\alpha,x,1) &= p(\alpha,x,1)-b_u \\ &- \phi_t\beta\int\int \max\{S_{st}(\alpha,x,1),S_{st}(\alpha,x,0),0\}d^2F(\alpha,x) \\ &+ (\lambda_{0y}+\lambda_{1y})\ell_{it}S_{it}(\alpha,x)+\lambda_{1y}\ell_{it}(U_{it}-U_{st}) \\ &+ \nu\max\{S_{st}(\alpha,x,0),S_{st}(\alpha,x,1),0\} \\ &+ \dot{S}_{st}(\alpha,x,1)\end{aligned} \tag{5}$$

if $m = 1$. Notice that for some (α, x), it might be the case that $S_{st}(\alpha, x, 0) > S_{st}(\alpha, x, 1)$. Differently than recovered, a match with a susceptible employee might optimally set $m = 0$ and produce only through home-working.

A.2 Dynamics of Baseline Model

The evolution of the measure of unemployed workers follows dynamic system given below where the first subindex denotes the health status $H \in \{s, i, r\}$ and the second the time t.

$$\begin{aligned}\dot{u}_{st} &= \psi+\delta\int\int e_{st}(\alpha,x)d\alpha dx+\nu\int\int e_{st}(\alpha,x)\{S_{st}(\alpha,x)<0\}d\alpha dx \\ &\quad -\phi_t\int\int\{S_{st}(\alpha,x)\geq 0\}d^2F(\alpha,x)u_{st}-\lambda_0\ell_{it}u_{st}-\eta u_{st} \\ \dot{u}_{it} &= \delta\int\int e_{it}(\alpha,x)d\alpha dx+\nu\int\int e_{it}(\alpha,x)\{S_{it}(\alpha,x)<0\}d\alpha dx \\ &\quad +\lambda_0\ell_{it}u_{st}-(\rho+\gamma(\ell_{it})+\eta)u_{it} \\ \dot{u}_{rt} &= \delta\int\int e_{rt}(\alpha,x)d\alpha dx+\nu\int\int e_{rt}(\alpha,x)\{S_{rt}(\alpha,x)<0\}d\alpha dx \\ &\quad +\rho u_{it}-\phi_t\int\int\{S_{rt}(\alpha,x)\geq 0\}d^2F(\alpha,x)u_{rt}-\eta u_{rt}\end{aligned}$$

For measures of employed, we also need to keep track of their match quality (α, x) and for the susceptible whether they work at home or away from home, taking subindex zero and one, respectively. Note the total susceptible employed in match (α, x) is the sum of those employed in that match working from home and outside of the home, $e_{st}(\alpha, x) := e_{0st}(\alpha, x) + e_{1st}(\alpha, x)$.

$$
\begin{aligned}
\dot{e}_{0st}(\alpha, x) = \; & u_{st}\phi_t\{S_{st}(\alpha, x) \geq 0\}f(\alpha, x) - (\delta + \eta)e_{0st}(\alpha, x) \\
& - \nu e_{0st}(\alpha, x)\{S_{st}(\alpha, x) < 0\} - \nu e_{0st}(\alpha, x)\{S_{st}(\alpha, x) \geq 0\}\{S_{st}(\alpha, x, 1) \geq S_{st}(\alpha, x, 0)\} \\
& + \nu e_{1st}(\alpha, x)\{S_{st}(\alpha, x) \geq 0\}\{S_{st}(\alpha, x, 1) < S_{st}(\alpha, x, 0)\} \\
& - e_{0st}(\alpha, x)\lambda_0 \ell_{it} \\
\dot{e}_{1st}(\alpha, x) = \; & u_{st}\phi_t\{S_{st}(\alpha, x) \geq 0\}f(\alpha, x) - (\delta + \eta)e_{1st}(\alpha, x) \\
& - \nu e_{1st}(\alpha, x)\{S_{st}(\alpha, x) < 0\} - \nu e_{1st}(\alpha, x)\{S_{st}(\alpha, x) \geq 0\}\{S_{st}(\alpha, x, 1) < S_{st}(\alpha, x, 0)\} \\
& + \nu e_{0st}(\alpha, x)\{S_{st}(\alpha, x) \geq 0\}\{S_{st}(\alpha, x, 1) \geq S_{st}(\alpha, x, 0)\} \\
& - e_{1st}(\alpha, x)(\lambda_0 + \lambda_1)\ell_{it} \\
\dot{e}_{it}(\alpha, x) = \; & e_{0st}(\alpha, x)\lambda_0 \ell_{it} + e_{1st}(\alpha, x)(\lambda_0 + \lambda_1)\ell_{it} \\
& - \nu e_{it}(\alpha, x)\{S_{it}(\alpha, x) < 0\} - (\delta + \rho + \gamma(\ell_{it}) + \eta)e_{it}(\alpha, x) \\
\dot{e}_{rt}(\alpha, x) = \; & u_{rt}\phi_t\{S_{rt}(\alpha, x) \geq 0\}f(\alpha, x) + \rho e_{it}(\alpha, x) \\
& - (\delta + \eta)e_{rt}(\alpha, x) - \nu e_{rt}(\alpha, x)\{S_{rt}(\alpha, x) < 0\}
\end{aligned}
$$

The measures of retired evolve as follows:

$$
\begin{aligned}
\dot{o}_{st} &= \eta \left(u_{st} + \int\int (e_{0st}(\alpha, x) + e_{1st}(\alpha, x))\, d\alpha dx \right) - (\lambda_0 \ell_{it} + \chi)\, o_{st} \\
\dot{o}_{it} &= \eta \left(u_{it} + \int\int e_{it}(\alpha, x) d\alpha dx \right) + \lambda_0 \ell_{it} o_{st} - (\gamma_R(\ell_{it}) + \chi + \rho)\, o_{it} \\
\dot{o}_{rt} &= \eta \left(u_{rt} + \int\int e_{rt}(\alpha, x) d\alpha dx \right) + \rho o_{st} - \chi o_{rt}
\end{aligned}
$$

Finally, the infection rate evolves as:

$$\ell_{it} = \dot{L}_{it} - \dot{L}_t$$

where

$$\dot{L}_{it} = \dot{u}_{it} + \int\int \dot{e}_{it}(\alpha, x) d\alpha dx + \dot{o}_{it} \qquad \dot{L}_t = \sum_{h \in \{s,i,r\}} \left(\dot{u}_{ht} + \int\int \dot{e}_{ht}(\alpha, x) d\alpha dx + \dot{o}_{ht} \right)$$

As discussed in the main body of the text the economy is initiated from a pre-Covid steady state. That is setting the left hand side of the differential equations above and ℓ_{it} to zero.

This yields the following initial allocation. Where the superscript ss denotes steady state levels.

$$\begin{aligned} u_s^{ss} =& \frac{\psi(\delta+\eta)}{\delta\eta + \eta\phi^{ss}\int\int f(\alpha,x)\{S_s^{ss}(\alpha,x)\geq 0\}d\alpha dx + \eta^2} \\ e_s^{ss}(\alpha,x) =& \frac{u_s^{ss}\phi^{ss} f(\alpha,x)\{S_s^{ss}(\alpha,x)\geq 0\}}{\delta+\eta} = e_{1s}^{ss}(\alpha,x) \\ o_s^{ss} =& \frac{\psi}{\chi} \end{aligned}$$

A.3 Computational Algorithm

To solve the model we need to solve for the surplus functions denoted as $S_{st}(\alpha, x, m)$. For example, the value of a recovered individual, who will always opt to work outside of the home, yields a surplus given by

$$\begin{aligned} (r+\delta+\eta)S_{rt}(\alpha,x) = p(\alpha,x,1) - b_u - \phi_t\beta \int\int \max\{S_{rt}(\alpha,x),0\}d^2F(\alpha,x) \\ + \nu\left(\max\{S_{rt}(\alpha,x),0\} - S_{rt}(\alpha,x)\right) + \dot{S}_{rt}(\alpha,x). \end{aligned}$$

For this surplus function and all others we approximate the state of the economy at time t by the aggregate state vector $\Omega_t := (u_{st}, u_{rt}, \ell_{it})$ such that, for an arbitrary state Ω,

$$\begin{aligned} (r+\delta+\eta)S_r(\alpha,x;\Omega) = p(\alpha,x,1) - b_u - \phi(\Omega)\beta \int\int \max\{S_r(\alpha,x;\Omega),0\}d^2F(\alpha,x) \\ + \nu\left(\max\{S_r(\alpha,x;\Omega),0\} - S_r(\alpha,x;\Omega)\right). \end{aligned}$$

Given the surplus functions, the transitional dynamics and the free entry condition defining $\phi(\Omega)$ can be computed exactly.[13] The solution algorithm works as follows.

- Construct a grid for five state variables, (α, x, Ω), where $\Omega := (u_s, u_r, L_i/L)$
- Guess $\phi^\star(\Omega)$
    - Solve fixed point for $S_r(\alpha, x; \Omega)$

[13] The omission of the continuation value $\dot{S}_{ht}(\alpha, x, m)$ omits an equilibrium effect from the model. That is a susceptible worker would rather become infected at the start of the pandemic, as their outside option of catching it then is smaller as they are still fairly likely in becoming infected. Thus the incentive for susceptible workers to self-isolate increases as the pandemic progresses. Interestingly, this is the opposite of the *behavioral* argument put forward by British scientists that warned that starting the lockdown earlier could lead to fatigue and less compliance later on. While interesting this paper abstracts from this mechanism.

- Solve fixed point for $S_i(\alpha, x; \Omega)$
- Solve fixed point (jointly) for $S_s(\alpha, x, 0; \Omega)$ and $S_s(\alpha, x, 1; \Omega)$
- Update $\phi^{\star}(\Omega)$ using free entry. Return to update surplus functions.

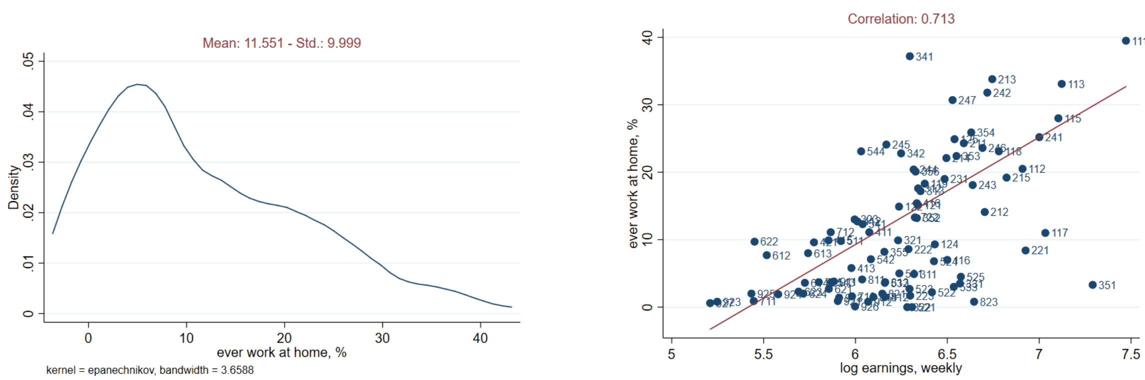
The model is solved for 50 grid points for x and α and ten for each of the aggregate states giving $(50^2 \times 10^3) = 2,500,000$ in total. After solutions are found for surpluses and job offer arrival rates the differential equations defining the aggregate states are approximated at a daily frequency.

A.4 Home working hours and earnings

Figure A.4: Home working hours and earnings

(a) Home working distribution

(b) Home working and log-earnings



Data are taken from the Annual Population Survey (APS), a survey of a representative sample of UK residents. Selected people are asked a number of questions about their relationship with the labour market, including questions on the extent to which they work from home. In particular, the current analysis exploits the answer reported by respondents to the following question: "Have you ever worked at home for your main job?". Data are then aggregated at occupational level using 3-digit codes (94 occupations in total). Figure A.4 panel (a) displays the distribution of employed workers who reported to ever worked at home, while panel (b) scatters the average share of workers reporting to work from home in each occupation against the average wage (panel b). We exploits this data in the calibration. Specifically, we target mean and standard deviation in the distribution of home-working respondents across occupations and the correlation between home-working shares and wages.

Online Appendix of
Twin Peaks: Covid-19 and the Labor Market

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1 Solving the model under lockdown

In the notation that follows, variables with a superscript L denote variables pertaining to the model under lockdown. Those without such superscripts refer to variables coming from the baseline model without lockdown described in Section 2 of Bradley et al. (2020).

1.1 Surplus functions of the lockdown model

This section proceeds by listing and describing the value functions associated with workers and vacancies of particular types. We then derive the corresponding surplus equations.

Retired workers

A retired worker that has recovered has value that comes from four sources. The first is their flow benefit of retirement. The second is the value associated with death by natural causes. The third is their value associated with the lockdown period ending, in which case they receive the baseline value of their status. The fourth is their continuation value. The formal representation follows

$$rR^L_{rt} = b_o + \chi(0 - R^L_{rt}) + \Lambda(R_r - R^L_{rt}) + \dot{R}^L_{rt}$$

An infected retired worker faces five terms associated with their value function. In addition to their flow benefit and continuation value, they face the value changes associated with the possibilities of death by natural causes, death by COVID-19, recovery and the economy leaving lockdown. The expression is given by

$$rR^L_{it} = b_o + (\chi + \gamma_o(\ell_{it}))(0 - R^L_{it}) + \rho_o(R^L_{rt} - R^L_{it}) + \Lambda(R_{it} - R^L_{it}) + \dot{R}^L_{it}$$

A retired worker that is susceptible receives a flow value, continuation value, faces a possibility of death and infection and the changes from the economy leaving lockdown. The value function can be written as

$$rR^L_{st} = b_o + \chi(0 - R^L_{st}) + \lambda^L_0 \ell_{it}(R^L_{it} - R^L_{st}) + \Lambda(R_{st} - R^L_{st}) + \dot{R}^L_{st}.$$

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Recovered young workers

The value of unemployment for a young worker is comprised of six terms. The first, the flow benefit of unemployment, is the same as in the baseline model. Their option value of finding work is now split into two terms, capturing the two possibilities of matching with a firm in the locked and unlocked sectors. The worker can retire at the rate η and the economy can come out of lockdown, in which case the worker receives the value of being recovered in the baseline model. They also receive their continuation value. The formal representation is given below

$$\begin{aligned} rU_{rt}^L = b_u &+ \phi_t \beta \pi \int\int \max\{S_{rt}^L(\alpha, x; L), 0\} d^2F(\alpha, x) \\ &+ \phi_t \beta(1-\pi) \int\int \max\{S_{rt}^L(\alpha, x; U), 0\} d^2F(\alpha, x) \\ &+ \eta(R_{rt}^L - U_{rt}^L) + \Lambda(U_{rt} - U_{rt}^L) + \dot{U}_{rt}^L \end{aligned}$$

A recovered worker who is employed now has two separate values, depending on whether their job is in the locked or unlocked sector. A worker with match characteristics (α, x) has contract with wage w and stipulation $m \in \{0, 1\}$ where 1 means working away from home and 0 means working from home. If the match is in the locked sector, then the value for being employed is given by

$$\begin{aligned} rW_{rt}^L(w, \alpha, x, m; L) = w &+ \delta(U_{rt}^L - W_{rt}^L(w, \alpha, x, m; L)) + \eta(R_{rt}^L - W_{rt}^L(w, \alpha, x, m; L)) \\ &+ \nu \left(\beta \max\{S_{rt}^L(\alpha, x, 0; L), S_{rt}^L(\alpha, x, 1; L), 0\} + U_{rt}^L - W_{rt}^L(w, \alpha, x; L)\right) \\ &+ \Lambda(W_{rt}(w, \alpha, x, m) - W_{rt}^L(w, \alpha, x, m; L)) + \dot{W}_{rt}^L(w, \alpha, x, m; L) \end{aligned}$$

where the argument L inside the parentheses signifies that the job is in the locked sector. Notice that, although the working arrangement regarding m can be negotiated in the contract, a match that is locked is unable to operate away from home for the duration of the lockdown. However, once the lockdown ends, we assume that they immediately start producing using the contracted arrangement regarding m. A match that is unlocked has the value

$$\begin{aligned} rW_{rt}^L(w, \alpha, x, m; U) = w &+ \delta(U_{rt}^L - W_{rt}^L(w, \alpha, x, m; U)) + \eta(R_{rt}^L - W_{rt}^L(w, \alpha, x, m; U)) \\ &+ \nu \left(\beta \max\{S_{rt}^L(\alpha, x, 0; U), S_{rt}^L(\alpha, x, 1; L), 0\} + U_{rt}^L - W_{rt}^L(w, \alpha, x; U)\right) \\ &+ \Lambda(W_{rt}(w, \alpha, x, m) - W_{rt}^L(w, \alpha, x, m; U)) + \dot{W}_{rt}^L(w, \alpha, x, m; U), \end{aligned}$$

where this match will involve working away from home during the lockdown when the contract has $m = 1$.

Value of a filled vacancy with a recovered worker

The value of a filled job is affected directly by whether or not the job is locked. A locked job has flow value that comes from home production less the wages paid to the employee. The match will break when either the exogenous separation shock is realised, or if the worker retires. It has option value associated with re-negotiation as well as with the economy leaving lockdown; there is also an associated continuation value. The formal expression is given by

$$\begin{aligned} rJ^L_{rt}(w, \alpha, x, m; L) &= p(\alpha, x, 0) - w + (\delta + \eta)(V^L_t - J^L_{rt}(w, \alpha, x, m; L)) \\ &\quad + \nu \left((1-\beta)\max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} + V^L_t - J^L_{rt}(w, \alpha, x, m; L)\right) \\ &\quad + \Lambda(J_{rt}(w, \alpha, x, m) - J^L_{rt}(w, \alpha, x, m; L)) + \dot{J}^L_{rt}(w, \alpha, x, m; L). \end{aligned}$$

where notice that V^L_t denotes the value of a vacancy under lockdown. A match that's in the unlocked sector may either be producing from home or away from home depending on the characteristics of the match; it faces no restriction. The value function for an unlocked match for contract stipulation m is

$$\begin{aligned} rJ^L_{rt}(w, \alpha, x, m; U) &= p(\alpha, x, m) - w + (\delta + \eta)(V^L_t - J^L_{rt}(w, \alpha, x, m; U)) \\ &\quad + \nu \left((1-\beta)\max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} + V^L_t - J^L_{rt}(w, \alpha, x, m; U)\right) \\ &\quad + \Lambda(J_{rt}(w, \alpha, x, m) - J^L_{rt}(w, \alpha, x, m; U)) + \dot{J}^L_{rt}(w, \alpha, x, m; U) \end{aligned}$$

where notice that the match's output varies with m.

Surplus of a match with a recovered worker

Imposing the equilibrium free entry condition, that $V^L_t = 0$, then combining expressions for retired workers, young workers and filled vacancy value functions gives the surplus equation

$$\begin{aligned} (r + \delta + \eta + \nu + \Lambda)S^L_{rt}(\alpha, x, m; L) &= p(\alpha, x, 0) - b_u \\ &\quad - \phi_t \beta \pi \int\int \max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} d^2F(\alpha, x) \\ &\quad - \phi_t \beta(1-\pi) \int\int \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} d^2F(\alpha, x) \\ &\quad + \nu \max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} \\ &\quad + \Lambda S_{rt}(\alpha, x, m) + \dot{S}^L_{rt}(\alpha, x, m; L) \end{aligned}$$

where again notice that the output comes from home production irrespective of the contractual choice of $m \in \{0, 1\}$. Then the surplus for an unlocked match with m is given by

$$
\begin{aligned}
(r+\delta+\eta+\nu+\Lambda)S^L_{rt}(\alpha,x,m;U) &= p(\alpha,x,m) - b_u \\
&- \phi_t\beta\pi\int\int \max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L),0\}d^2F(\alpha,x) \\
&- \phi_t\beta(1-\pi)\int\int \max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U),0\}d^2F(\alpha,x) \\
&+ \nu\max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U),0\} \\
&+ \Lambda S_{rt}(\alpha,x,m) + \dot{S}^L_{rt}(\alpha,x,m;U).
\end{aligned}
$$

Infected young workers

An unemployed infected worker's value function contains six terms. They receive their flow value of unemployment and continuation value. They receive value associated with the possibility of moving to recovered status, from dying from the virus, from retiring and from the economy leaving the state of lockdown. The formal expression is given by

$$
rU^L_{it} = b_u + \rho_y\left(U^L_{rt} - U^L_{it}\right) + \gamma_{0y}(\ell_{it})(0 - U^L_{it}) + \eta(R^L_{it} - U^L_{it}) + \Lambda(U_{it} - U^L_{it}) + \dot{U}^L_{it}.
$$

An employed worker with infection status is at home on sick pay. They produce no output but continue to receive their contracted-upon wage; upon recovery, they return back to production for the firm. Consequently, whether the infected worker's match is in the locked or unlocked sector affects their value function. A worker in the locked sector receives value from their wage, continuation value, value associated with recovery, death, retirement, renegotiation and lockdown being lifted. The expression is as follows

$$
\begin{aligned}
rW^L_{it}(w,\alpha,x;L) &= w + \rho_y\left(W^L_{rt}(w,\alpha,x,1;L) - W^L_{it}(w,\alpha,x;L)\right) \\
&+ \gamma_{0y}(\ell_{it})(0 - W^L_{it}(w,\alpha,x;L)) \\
&+ \delta\left(U^L_{it} - W^L_{it}(w,\alpha,x;L)\right) + \eta\left(R^L_{it} - W^L_{it}(w,\alpha,x;L)\right) \\
&+ \nu\left(\beta\max\{S^L_{it}(\alpha,x;L),0\} + U^L_{it} - W^L_{it}(w,\alpha,x;L)\right) \\
&+ \Lambda(W_{it}(w,\alpha,x) - W^L_{it}(w,\alpha,x;L)) + \dot{W}^L_{it}(w,\alpha,x;L)
\end{aligned}
$$

and similarly for an unlocked infected worker, the expression is

$$
\begin{aligned}
rW^L_{it}(w,\alpha,x;U) &= w + \rho_y\left(W^L_{rt}(w,\alpha,x,1;U) - W^L_{it}(w,\alpha,x;U)\right) \\
&+ \gamma_{0y}(\ell_{it})(0 - W^L_{it}(w,\alpha,x;U)) \\
&+ \delta\left(U^L_{it} - W^L_{it}(w,\alpha,x;U)\right) + \eta\left(R^L_{it} - W^L_{it}(w,\alpha,x;U)\right) \\
&+ \nu\left(\beta\max\{S^L_{it}(\alpha,x;U),0\} + U^L_{it} - W^L_{it}(w,\alpha,x;U)\right) \\
&+ \Lambda(W_{it}(w,\alpha,x) - W^L_{it}(w,\alpha,x;U)) + \dot{W}^L_{it}(w,\alpha,x;U).
\end{aligned}
$$

One point to note is that the match retains its status with regard to being in the locked or unlocked sector throughout the health status changes of the worker. That is — whatever sector their match belonged to prior and during infection — the match will remain in that sector subsequent to recovery.

Value of a filled vacancy with an infected worker

A firm that has an infected worker pays their wage as sick pay for the duration of their illness in the absence of separation. The firm receives the value associated with the possibility of the worker's recovery and the match can be broken through either exogenous or endogenous separation at re-negotiation, retirement or through death of the worker from the virus. The formal expression for a match with an infected worker in a locked sector is given by

$$\begin{aligned} rJ^L_{it}(w,\alpha,x;L) &= -w + \rho_y \left(J^L_{rt}(w,\alpha,x,1;L) - J^L_{it}(w,\alpha,x;L)\right) \\ &\quad + (\gamma_{0y}(\ell_{it}) + \delta + \eta)(V^L_t - J^L_{it}(w,\alpha,x;L)) \\ &\quad + \nu\left((1-\beta)\max\{S^L_{it}(\alpha,x;L),0\} + V^L_t - J^L_{it}(w,\alpha,x;L)\right) \\ &\quad + \Lambda(J_{it}(w,\alpha,x) - J^L_{it}(w,\alpha,x;L)) + \dot{J}^L_{it}(w,\alpha,x;L) \end{aligned}$$

while that in the unlocked sector is

$$\begin{aligned} rJ^L_{it}(w,\alpha,x;U) &= -w + \rho_y \left(J^L_{rt}(w,\alpha,x,1;U) - J^L_{it}(w,\alpha,x;U)\right) \\ &\quad + (\gamma_{0y}(\ell_{it}) + \delta + \eta)(V^L_t - J^L_{it}(w,\alpha,x;U)) \\ &\quad + \nu\left((1-\beta)\max\{S^L_{it}(\alpha,x;U),0\} + V^L_t - J^L_{it}(w,\alpha,x;U)\right) \\ &\quad + \Lambda(J_{it}(w,\alpha,x) - J^L_{it}(w,\alpha,x;U)) + \dot{J}^L_{it}(w,\alpha,x;U) \end{aligned}$$

Surplus of a match with an infected worker

The surplus can be found through using the value functions for employment, unemployment and value of a filled job for the locked and unlocked sectors to get the surplus. Using the equilibrium condition that the value to a vacancy is zero, the surplus for a locked match is

$$\begin{aligned} (r + \rho_y + \gamma_{0y}(\ell_{it}) + \delta + \eta + \nu + \Lambda)S^L_{it}(\alpha,x;L) &= -b_u + \rho_y S^L_{rt}(\alpha,x;L) \\ &\quad + \nu\max\{S^L_{it}(\alpha,x;L),0\} + \Lambda S_{it}(\alpha,x) + \dot{S}^L_{it}(\alpha,x;L) \end{aligned}$$

while that for an unlocked match is

$$\begin{aligned} (r + \rho_y + \gamma_{0y}(\ell_{it}) + \delta + \eta + \nu + \Lambda)S^L_{it}(\alpha,x;U) &= -b_u + \rho_y S^L_{rt}(\alpha,x;U) \\ &\quad + \nu\max\{S^L_{it}(\alpha,x;U),0\} + \Lambda S_{it}(\alpha,x) + \dot{S}^L_{it}(\alpha,x;U). \end{aligned}$$

Susceptible young workers

The value to being an unemployed susceptible worker closely resembles that of a recovered worker, with the exception of an additional value change associated with the possibility of infection. Their value function is given as

$$\begin{aligned} rU_{st}^L = b_u &+ \phi_t \pi \beta \int\int \max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L), 0\} d^2F(\alpha,x) \\ &+ \phi_t(1-\pi)\beta \int\int \max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U), 0\} d^2F(\alpha,x) \\ &+ \lambda_0^L \ell_{it}(U_{it}^L - U_{st}^L) + \eta(R_{st}^L - U_{st}^L) + \Lambda(U_{st} - U_{st}^L) + \dot{U}_{st}^L \end{aligned}$$

where notice that the rate of infection is given by $\lambda_0^L \ell_{it}$, using the lockdown infection parameter that exists regardless of working decisions. A worker that is employed with contract for wages and working arrangements (w, m) has value that differs based on the sector they work in. When locked, the worker's value function is given by

$$\begin{aligned} rW_{st}^L(w,\alpha,x,m;L) = w &+ \delta(U_{st}^L - W_{st}^L(w,\alpha,x,m;L)) + \eta(R_{st}^L - W_{st}^L(w,\alpha,x,m;L)) \\ &+ \lambda_0^L \ell_{it} \left(W_{it}^L(w,\alpha,x,L) - W_{st}^L(w,\alpha,x,m;L)\right) \\ &+ \nu\left(\beta \max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L), 0\} + U_{st}^L - W_{st}^L(w,\alpha,x,m;L)\right) \\ &+ \Lambda(W_{st}(w,\alpha,x,m) - W_{st}^L(w,\alpha,x,m;L)) + \dot{W}_{st}^L(w,\alpha,x,m;L) \end{aligned}$$

where notice that their infection rate is the same as that of the unemployed worker given that they are unable to work away from home. In contrast, a worker in an unlocked sector has value functions that differ based on the contracted m. For $m = 0$, see that

$$\begin{aligned} rW_{st}^L(w,\alpha,x,0;U) = w &+ \delta(U_{st}^L - W_{st}^L(w,\alpha,x,0;U)) + \eta(R_{st}^L - W_{st}^L(w,\alpha,x,0;U)) \\ &+ \lambda_0^L \ell_{it} \left(W_{it}^L(w,\alpha,x,U) - W_{st}^L(w,\alpha,x,0;U)\right) \\ &+ \nu\left(\beta \max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U), 0\} + U_{st}^L - W_{st}^L(w,\alpha,x,0;U)\right) \\ &+ \Lambda(W_{st}(w,\alpha,x,0) - W_{st}^L(w,\alpha,x,0;U)) + \dot{W}_{st}^L(w,\alpha,x,0;U) \end{aligned}$$

and then for $m = 1$

$$\begin{aligned} rW_{st}^L(w,\alpha,x,1;U) = w &+ \delta(U_{st}^L - W_{st}^L(w,\alpha,x,1;U)) + \eta(R_{st}^L - W_{st}^L(w,\alpha,x,1;U)) \\ &+ \left[\lambda_0^L + \lambda_1\right] \ell_{it} \left(W_{it}^L(w,\alpha,x,U) - W_{st}^L(w,\alpha,x,1;U)\right) \\ &+ \nu\left(\beta \max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U), 0\} + U_{st}^L - W_{st}^L(w,\alpha,x,1;U)\right) \\ &+ \Lambda(W_{st}(w,\alpha,x,0) - W_{st}^L(w,\alpha,x,1;U)) + \dot{W}_{st}^L(w,\alpha,x,1;U) \end{aligned}$$

where the distinction between the two is the higher infection rate when working away from home.

Value of a filled vacancy with an susceptible worker

The value to a filled job with a susceptible worker in a locked industry for arbitrary contract (w, m) is given as follows

$$\begin{aligned} rJ_{st}^{L}(w,\alpha,x,m;L) &= p(\alpha,x,0) - w + (\delta+\eta)(V_t^L - J_{st}^L(w,\alpha,x,m;L)) \\ &+ \lambda_0^L \ell_{it}\left(J_{it}^L(w,\alpha,x;L) - J_{st}^L(w,\alpha,x,m;L)\right) \\ &+ \nu\left((1-\beta)\max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L),0\} + V_t^L - J_{st}^L(w,\alpha,x,m;L)\right) \\ &+ \Lambda(J_{st}(w,\alpha,x,m) - J_{st}^L(w,\alpha,x,m;L)) + \dot{J}_{st}(w,\alpha,x,m;L). \end{aligned}$$

A match that is unlocked differs based on locked status. An unlocked match with $m = 0$ delivers value of

$$\begin{aligned} rJ_{st}^{L}(w,\alpha,x,0;U) &= p(\alpha,x,0) - w + (\delta+\eta)(V_t^L - J_{st}^L(w,\alpha,x,0;U)) \\ &+ \lambda_0^L \ell_{it}\left(J_{it}^L(w,\alpha,x;U) - J_{st}^L(w,\alpha,x,0;U)\right) \\ &+ \nu\left((1-\beta)\max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U),0\} + V_t^L - J_{st}^L(w,\alpha,x,0;U)\right) \\ &+ \Lambda(J_{st}(w,\alpha,x,0) - J_{st}^L(w,\alpha,x,0;U)) + \dot{J}_{st}(w,\alpha,x,0;U). \end{aligned}$$

and that for an $m = 1$ contract gives

$$\begin{aligned} rJ_{st}^{L}(w,\alpha,x,1;U) &= p(\alpha,x,1) - w + (\delta+\eta)(V_t^L - J_{st}^L(w,\alpha,x,1;U)) \\ &+ [\lambda_0^L + \lambda_1]\ell_{it}\left(J_{it}^L(w,\alpha,x;U) - J_{st}^L(w,\alpha,x,1;U)\right) \\ &+ \nu\left((1-\beta)\max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U),0\} + V_t^L - J_{st}^L(w,\alpha,x,1;U)\right) \\ &+ \Lambda(J_{st}(w,\alpha,x,1) - J_{st}^L(w,\alpha,x,1;U)) + \dot{J}_{st}(w,\alpha,x,1;U). \end{aligned}$$

where the match now produces the higher away from home level of output, in addition to the rate of change to infected status being higher by λ_1.

Surplus of a match with a susceptible worker

The surplus from a match in the locked sector for $m \in \{0, 1\}$ is given as

$$\begin{aligned} (r+\delta+\eta+\lambda_0^L\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,m;L) &= p(\alpha,x,0) - b_u \\ &- \phi_t\pi\beta\int\int\max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L),0\}d^2F(\alpha,x) \\ &- \phi_t(1-\pi)\beta\int\int\max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U),0\}d^2F(\alpha,x) \\ &+ \lambda_0^L\ell_{it}S_{it}^L(\alpha,x;L) + \nu\max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L),0\} \\ &+ \Lambda S_{st}(\alpha,x,m) + \dot{S}_{st}^L(\alpha,x,m;L). \end{aligned}$$

The surplus for a match in the unlocked sector with $m = 0$ is

$$
\begin{aligned}
(r+\delta+\eta+\lambda_0^L \ell_{it}+\nu+\Lambda) S_{st}^L(\alpha, x, 0; U) = {} & p(\alpha, x, 0) - b_u \\
& - \phi_t \pi \beta \int\int \max\{S_{st}^L(\alpha, x, 0; L), S_{st}^L(\alpha, x, 1; L), 0\} d^2F(\alpha, x) \\
& - \phi_t (1-\pi) \beta \int\int \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} d^2F(\alpha, x) \\
& + \lambda_0^L \ell_{it} S_{it}^L(\alpha, x; U) + \nu \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} \\
& + \Lambda S_{st}(\alpha, x, 0) + \dot{S}_{st}^L(\alpha, x, 0; U)
\end{aligned}
$$

and that for the unlocked sector with $m = 1$ is

$$
\begin{aligned}
(r+\delta+\eta+(\lambda_0^L+\lambda_1)\ell_{it}+\nu+\Lambda) S_{st}^L(\alpha, x, 1; U) = {} & p(\alpha, x, 1) - b_u \\
& - \phi_t \pi \beta \int\int \max\{S_{st}^L(\alpha, x, 0; L), S_{st}^L(\alpha, x, 1; L), 0\} d^2F(\alpha, x) \\
& - \phi_t (1-\pi) \beta \int\int \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} d^2F(\alpha, x) \\
& + (\lambda_0^L+\lambda_1) \ell_{it} S_{it}^L(\alpha, x; U) + \lambda_1 \ell_{it} (U_{it}^L - U_{st}^L) \\
& + \nu \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} \\
& + \Lambda S_{st}(\alpha, x, 1) + \dot{S}_{st}^L(\alpha, x, 1; U).
\end{aligned}
$$

1.2 Vacant jobs

Vacant jobs get in contact with unemployed workers at a rate ϕ_t^f. Upon contracting, the firm receives fraction $(1-\beta)$ of the match's generated surplus. Notice that there are four possibilities for a given vacancy with regard to the type of match that is formed. The potential unemployed workers they can match with differ along the health dimension — they could either be susceptible or recovered. In addition, there are two possibilities from the perspective of the production being in either the locked or unlocked sectors. As such, there are five terms in the value to a vacancy: the flow cost κ as well as terms capturing these four possibilities

$$
\begin{aligned}
rV_t^L = -\kappa &+ \phi_t^f(1-\beta)\frac{u_{st}^L}{u_{st}^L+u_{rt}^L}\pi\int\int\max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}dF^2(\alpha,x)\\
&+ \phi_t^f(1-\beta)\frac{u_{st}^L}{u_{st}^L+u_{rt}^L}(1-\pi)\int\int\max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}dF^2(\alpha,x)\\
&+ \phi_t^f(1-\beta)\frac{u_{rt}^L}{u_{st}^L+u_{rt}^L}\pi\int\int\max\{S_{rt}^L(\alpha,x;L),0\}dF^2(\alpha,x)\\
&+ \phi_t^f(1-\beta)\frac{u_{rt}^L}{u_{st}^L+u_{rt}^L}(1-\pi)\int\int\max\{S_{rt}^L(\alpha,x;U),0\}dF^2(\alpha,x)
\end{aligned}
$$

where u_{st}^L and u_{rt}^L denote the measures of unemployed workers with susceptible and recovered status under the lockdown model.

2 Dynamics of the lockdown model

This section details the dynamics of the measures of workers in differing employment, age and health statuses in the model with lockdown. Measures with L superscripts correspond to those under lockdown, while those without are from the baseline model, (out of lockdown). Assume that the lockdown commences at an arbitrary time denoted by $\bar{t}$. At this point, measures are divided-up as follows. The measures of the three different health statuses for unemployment will be equal to their pre-lockdown measures

$$
\begin{aligned}
u_{s\bar{t}}^L &= u_{s\bar{t}}\\
u_{i\bar{t}}^L &= u_{i\bar{t}}\\
u_{r\bar{t}}^L &= u_{r\bar{t}}
\end{aligned}
$$

which are the measures of susceptible, infected and recovered respectively. Similarly for the retired at the time of lockdown's commencement

$$
\begin{aligned}
o_{s\bar{t}}^L &= o_{s\bar{t}}\\
o_{i\bar{t}}^L &= o_{i\bar{t}}\\
o_{r\bar{t}} &= o_{r\bar{t}}
\end{aligned}
$$

which are the measures of retired people across the susceptible, infected and recovered health statuses respectively. The measures of employed workers of a given health status and match state (α, x) will be split such that fraction π will be placed into the locked sector while fraction $1-\pi$ will be in the unlocked sector as follows

$$\begin{aligned}
e^L_{0s\bar{t}}(\alpha, x; L) &= \pi e_{0s\bar{t}}(\alpha, x) \\
e^L_{0s\bar{t}}(\alpha, x; U) &= (1-\pi) e_{0s\bar{t}}(\alpha, x) \\
e^L_{1s\bar{t}}(\alpha, x; L) &= \pi e_{1s\bar{t}}(\alpha, x) \\
e^L_{1s\bar{t}}(\alpha, x; U) &= (1-\pi) e_{1s\bar{t}}(\alpha, x) \\
e^L_{r\bar{t}}(\alpha, x; L) &= \pi e_{r\bar{t}}(\alpha, x) \\
e^L_{r\bar{t}}(\alpha, x; U) &= (1-\pi) e_{r\bar{t}}(\alpha, x) \\
e^L_{i\bar{t}}(\alpha, x; L) &= \pi e_{i\bar{t}}(\alpha, x) \\
e^L_{i\bar{t}}(\alpha, x; U) &= (1-\pi) e_{i\bar{t}}(\alpha, x)
\end{aligned}$$

which respectively represent the measures of susceptible workers at home in locked jobs and unlocked jobs, of susceptible workers away from home in locked and unlocked jobs, of recovered workers in locked and unlocked jobs and infected workers in locked and unlocked jobs. From the point where the lockdown commences, these measures evolve endogenously. The laws of motion for the retired are

$$\begin{aligned}
\dot{o}^L_{st} = \eta &\left(u^L_{st} + \int\int e^L_{0st}(\alpha, x; L) d\alpha dx + \int\int e^L_{0st}(\alpha, x; U) d\alpha dx \right) \\
&+ \eta \left(\int\int e^L_{1st}(\alpha, x; L) d\alpha dx + \int\int e^L_{1st}(\alpha, x; U) d\alpha dx \right) - (\lambda^L_0 \ell^L_{it} + \chi) o^L_{st}
\end{aligned}$$

$$\begin{aligned}
\dot{o}^L_{it} = \eta &\left(u^L_{it} + \int\int e^L_{it}(\alpha, x; L) d\alpha dx + \int\int e^L_{it}(\alpha, x; U) d\alpha dx \right) \\
&+ \lambda^L_0 \ell^L_{it} o^L_{st} - (\gamma_R(\ell^L_{it}) + \chi + \rho_o) o^L_{it}
\end{aligned}$$

$$\begin{aligned}
\dot{o}^L_{rt} = \eta &\left(u^L_{rt} + \int\int e^L_{rt}(\alpha, x; L) d\alpha dx + \int\int e^L_{rt}(\alpha, x; U) d\alpha dx \right) \\
&+ \rho_o o^L_{it} - \chi o^L_{rt}
\end{aligned}$$

while those for the unemployed are

$$
\begin{aligned}
\dot{u}^L_{st} &= \psi + \delta \int\int e^L_{0st}(\alpha, x; L) d\alpha dx + \delta \int\int e^L_{1st}(\alpha, x; L) d\alpha dx \\
&+ \delta \int\int e^L_{0st}(\alpha, x; U) d\alpha dx + \delta \int\int e^L_{1st}(\alpha, x; U) d\alpha dx \\
&+ \nu \int\int e^L_{0st}(\alpha, x; L)\{S^L_{0st}(\alpha, x; L) < 0\} d\alpha dx + \nu \int\int e^L_{1st}(\alpha, x; L)\{S^L_{1st}(\alpha, x; L) < 0\} d\alpha dx \\
&+ \nu \int\int e^L_{0st}(\alpha, x; U)\{S^L_{0st}(\alpha, x; U) < 0\} d\alpha dx + \nu \int\int e^L_{1st}(\alpha, x; U)\{S^L_{1st}(\alpha, x; U) < 0\} d\alpha dx \\
&- \phi^L_t \pi \int\int \{S^L_{st}(\alpha, x, L) \geq 0\} d^2F(\alpha, x) u^L_{st} - \phi^L_t (1-\pi) \int\int \{S^L_{st}(\alpha, x, U) \geq 0\} d^2F(\alpha, x) u^L_{st} \\
&- \lambda^L_0 \ell^L_{it} u^L_{st} - \eta u^L_{st}
\end{aligned}
$$

$$
\begin{aligned}
\dot{u}^L_{it} &= \delta \int\int e^L_{it}(\alpha, x; L) d\alpha dx + \delta \int\int e^L_{it}(\alpha, x; U) d\alpha dx \\
&+ \nu \int\int e^L_{it}(\alpha, x; L)\{S^L_{it}(\alpha, x; L) < 0\} d\alpha dx + \nu \int\int e^L_{it}(\alpha, x; U)\{S^L_{it}(\alpha, x; U) < 0\} d\alpha dx \\
&+ \lambda^L_0 \ell^L_{it} u^L_{st} - (\rho_y + \gamma(\ell^L_{it}) + \eta) u^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
\dot{u}^L_{rt} &= \delta \int\int e^L_{rt}(\alpha, x; L) d\alpha dx + \delta \int\int e^L_{rt}(\alpha, x; U) d\alpha dx \\
&+ \nu \int\int e^L_{rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) < 0\} d\alpha dx + \nu \int\int e^L_{rt}(\alpha, x; U)\{S^L_{rt}(\alpha, x; U) < 0\} d\alpha dx \\
&+ \rho_y u^L_{it} - \phi^L_t \pi \int\int \{S^L_{rt}(\alpha, x, L) \geq 0\} d^2F(\alpha, x) u^L_{rt} - \phi^L_t (1-\pi) \int\int \{S^L_{rt}(\alpha, x, U) \geq 0\} d^2F(\alpha, x) u^L_{rt}.
\end{aligned}
$$

For the employed workers, we track measures across the different match characteristics and lock statuses. For measures of susceptible employed, the equations are as follows.

$$\begin{aligned}
\dot{e}^L_{0st}(\alpha,x;L) &= \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}f(\alpha,x)\\
&-(\delta+\eta)e^L_{0st}(\alpha,x;L)-\nu e^L_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x,0;L)<0\}\\
&-\nu e^L_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}\\
&+\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}\\
&-e^L_{0st}(\alpha,x;L)\lambda^L_0\ell^L_{it}
\end{aligned}$$

$$\begin{aligned}
\dot{e}^L_{0st}(\alpha,x;U) &= (1-\pi)u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\
&-(\delta+\eta)e^L_{0st}(\alpha,x;U)-\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x,0;U)<0\}\\
&-\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}\\
&+\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}\\
&-e^L_{0st}(\alpha,x;U)\lambda^L_0\ell^L_{it}
\end{aligned}$$

$$\begin{aligned}
\dot{e}^L_{1st}(\alpha,x;L) &= \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}f(\alpha,x)\\
&-(\delta+\eta)e^L_{1st}(\alpha,x;L)-\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x,1;L)<0\}\\
&-\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}\\
&+\nu e_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}\\
&-e^L_{1st}(\alpha,x;L)\lambda^L_0\ell^L_{it}
\end{aligned}$$

$$\begin{aligned}
\dot{e}^L_{1st}(\alpha,x;U) &= (1-\pi)u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\
&-(\delta+\eta)e^L_{1st}(\alpha,x;U)-\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x,1;U)<0\}\\
&-\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}\\
&+\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}\\
&-e^L_{1st}(\alpha,x;U)(\lambda^L_0+\lambda_1)\ell^L_{it}.
\end{aligned}$$

For the measures of infected employed, the dynamics evolve according to

$$\begin{aligned}
\dot{e}^L_{it}(\alpha,x;L) &= e^L_{0st}(\alpha,x;L)\lambda^L_0\ell^L_{it}+e^L_{1st}(\alpha,x;L)(\lambda^L_0)\ell^L_{it}-\nu e^L_{it}(\alpha,x;L)\{S^L_{it}(\alpha,x;L)<0\}\\
&-(\delta+\rho_y+\gamma(\ell^L_{it})+\eta)e^L_{it}(\alpha,x;L)
\end{aligned}$$

$$\begin{aligned}
\dot{e}^L_{it}(\alpha,x;U) &= e^L_{0st}(\alpha,x;U)\lambda^L_0\ell^L_{it}+e^L_{1st}(\alpha,x;U)(\lambda^L_0+\lambda_1)\ell^L_{it}-\nu e^L_{it}(\alpha,x;U)\{S^L_{it}(\alpha,x;U)<0\}\\
&-(\delta+\rho_y+\gamma(\ell^L_{it})+\eta)e^L_{it}(\alpha,x;U).
\end{aligned}$$

The measures of employed that are recovered evolve as follows

$$\dot{e}^L_{rt}(\alpha, x; L) = \pi u^L_{rt}\phi_t\{S^L_{rt}(\alpha, x; L) \geq 0\}f(\alpha, x) + \rho_y e^L_{it}(\alpha, x; L) - (\delta + \eta)e^L_{rt}(\alpha, x; L)$$
$$- \nu e^L_{rt}(\alpha, x)\{S^L_{rt}(\alpha, x; L) < 0\}$$

$$\dot{e}^L_{rt}(\alpha, x; U) = (1 - \pi)u^L_{rt}\phi^L_t\{S^L_{rt}(\alpha, x; U) \geq 0\}f(\alpha, x) + \rho_y e^L_{it}(\alpha, x; U) - (\delta + \eta)e^L_{rt}(\alpha, x; U)$$
$$- \nu e^L_{rt}(\alpha, x)\{S^L_{rt}(\alpha, x; U) < 0\}.$$

Finally, denote the time where lockdown ceases by $\hat{t}$. At this time the measure of unemployed of each status for non-lockdown are equal to their lockdown level as follows

$$u_{s\hat{t}} = u^L_{s\hat{t}}$$
$$u_{i\hat{t}} = u^L_{i\hat{t}}$$
$$u_{r\hat{t}} = u^L_{r\hat{t}}$$

for susceptible, infected and retired respectively. Similarly, for the three health statuses of retired workers

$$o_{s\hat{t}} = o^L_{s\hat{t}}$$
$$o_{i\hat{t}} = o^L_{i\hat{t}}$$
$$o_{r\hat{t}} = o^L_{r\hat{t}}.$$

Lastly, for each employment health status and idiosyncratic match state, the measures of locked and unlocked matches are summed to together as follows

$$e_{0s\hat{t}}(\alpha, x) = e^L_{0s\hat{t}}(\alpha, x; L) + e^L_{0s\hat{t}}(\alpha, x; U)$$
$$e_{1s\hat{t}}(\alpha, x) = e^L_{1s\hat{t}}(\alpha, x; L) + e^L_{1s\hat{t}}(\alpha, x; U)$$
$$e_{r\hat{t}}(\alpha, x) = e^L_{r\hat{t}}(\alpha, x; L) + e^L_{r\hat{t}}(\alpha, x; U)$$
$$e_{i\hat{t}}(\alpha, x) = e^L_{i\hat{t}}(\alpha, x; L) + e^L_{i\hat{t}}(\alpha, x; U)$$

which are for the susceptible employed working at home and away from home, the recovered and infected respectively. From time $\hat{t}$ onwards, the measures evolve endogenously as described in appendix **??**.

3 Wages under laissez-faire

Recovered individuals

For a recovered individual, an arbitrary wage leaves the employer with value equal to:

$$\begin{aligned}(r+\delta+\eta)J_{rt}(w,\alpha,x) &= p(\alpha,x,1)-w \\ &\quad + \nu\left((1-\beta)\max\{S_{rt}(\alpha,x),0\} - J_{rt}(w,\alpha,x)\right) + \dot{J}_{rt}(w,\alpha,x)\end{aligned}$$

Re-arranging terms, and substituting in the Nash splitting condition, $J_{rt}(w,\alpha,x) = (1-\beta)S_{rt}(\alpha,x)$, we get the following wage function for recovered employees:

$$\begin{aligned}w_{rt}(\alpha,x) &= p(\alpha,x,1) + \nu(1-\beta)\max\{S_{rt}(\alpha,x),0\} \\ &\quad - (r+\delta+\eta+\nu)(1-\beta)S_{rt}(\alpha,x) + (1-\beta)\dot{S}_{rt}(\alpha,x)\end{aligned}$$

Infected individuals

The infected never start new jobs (by assumption) and inherent their previous wages and are on sick leave. However, they can be hit by re-negotiation shock. Again for arbitrary wage,

$$\begin{aligned}rJ_{it}(w,\alpha,x) &= -w + \rho\left(J_{rt}(w,\alpha,x) - J_{it}(w,\alpha,x)\right) + (\gamma(\ell_{it})+\delta+\eta)(-J_{it}(w,\alpha,x)) \\ &\quad + \nu\left((1-\beta)\max\{S_{it}(\alpha,x),0\} - J_{it}(w,\alpha,x)\right) + \dot{J}_{it}(w,\alpha,x)\end{aligned}$$

re-arranging terms, and substituting again in the Nash splitting condition, $J_{it}(w,\alpha,x) = (1-\beta)S_{it}(\alpha,x)$, we obtain:

$$\begin{aligned}w_{it}(\alpha,x) &= \rho J_{rt}(w,\alpha,x) + \nu(1-\beta)\max\{S_{it}(\alpha,x),0\} \\ &\quad - (r+\rho+\gamma(\ell_{it})+\delta+\eta+\nu)(1-\beta)S_{it}(w,\alpha,x) + (1-\beta)\dot{S}_{it}(w,\alpha,x)\end{aligned}$$

There is a unique solution as derivative of lhs is negative. But it could be negative if component independent of wage is negative (and large). Since:

$$\begin{aligned}(r+\delta+\eta+\nu)J_{rt}(w_{it}(\alpha,x),\alpha,x) &= p(\alpha,x,1) - w_{it}(\alpha,x) \\ &\quad + \nu(1-\beta)\max\{S_{rt}(\alpha,x),0\} + \dot{J}_{rt}(w_{it}(\alpha,x),\alpha,x)\end{aligned}$$

then substituting:

$$\begin{aligned}w_{it}(\alpha,x)\frac{r+\delta+\eta+\nu+\rho}{r+\delta+\eta+\nu} &= \frac{\rho}{r+\delta+\eta+\nu}p(\alpha,x,1) + \frac{\rho\nu(1-\beta)}{r+\delta+\eta+\nu}\max\{S_{rt}(\alpha,x),0\} \\ &\quad + \frac{\rho}{r+\delta+\eta+\nu}\dot{J}_{rt}(w_{it}(\alpha,x),\alpha,x) \\ &\quad - (r+\rho+\gamma(\ell_{it})+\delta+\eta)(1-\beta)S_{it}(\alpha,x) + (1-\beta)\dot{S}_{it}(\alpha,x)\end{aligned}$$

Re-arranging terms, and using the Nash splitting condition, $J_{rt}(w, \alpha, x) = (1 - \beta)S_{rt}(\alpha, x)$, we get a final expression for the wage of infected employees:

$$\begin{aligned} w_{it}(\alpha, x) = & \frac{\rho}{r + \delta + \eta + \nu + \rho} p(\alpha, x, 1) + \frac{\rho\nu(1 - \beta)}{r + \delta + \eta + \nu + \rho} \max\{S_{rt}(\alpha, x), 0\} \\ & - \frac{(r + \delta + \eta + \nu)(r + \rho + \gamma(\ell_{it}) + \delta + \eta)}{r + \delta + \eta + \nu + \rho} (1 - \beta)S_{it}(\alpha, x) \\ & + \frac{\rho}{r + \delta + \eta + \nu + \rho} (1 - \beta)\dot{S}_{rt}(\alpha, x) \\ & + \frac{r + \delta + \eta + \nu}{r + \delta + \eta + \nu + \rho} (1 - \beta)\dot{S}_{it}(\alpha, x) \end{aligned}$$

Susceptible individuals

For arbitrary wage w, the value of a job (α, x) being worked from home by a susceptible individual is equal to:

$$\begin{aligned} (r + \delta + \eta + \lambda_0\ell_{it} + \nu)J_{st}(w, \alpha, x, 0) = & \, p(\alpha, x, 0) - w + \lambda_0\ell_{it}J_{it}(w, \alpha, x) \\ & + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\}) + \dot{J}_{st}(w, \alpha, x, 0) \end{aligned}$$

Re-arranging terms and using the Nash splitting conditions $J_{st}(w, \alpha, x, 0) = (1-\beta)S_{st}(w, \alpha, x, 0)$ and $J_{it}(w, \alpha, x) = (1 - \beta)S_{it}(w, \alpha, x)$, we can write the wage equation for susceptible employees only working at home as follows:

$$\begin{aligned} w_{st}(\alpha, x, 0) = & \, p(\alpha, x, 0) + \lambda_0\ell_{it}(1 - \beta)S_{it}(\alpha, x) \\ & - (r + \delta + \eta + \lambda_0\ell_{it} + \nu)(1 - \beta)S_{st}(\alpha, x, 0) \\ & + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\}) + (1 - \beta)\dot{S}_{st}(\alpha, x, 0) \end{aligned}$$

Similarly, the wage equation for susceptible employees working away from home is equal to:

$$\begin{aligned} w_{st}(\alpha, x, 1) = & \, p(\alpha, x, 1) + (\lambda_0 + \lambda_1)\ell_{it}(1 - \beta)S_{it}(\alpha, x) \\ & - (r + \delta + \eta + (\lambda_0 + \lambda_1)\ell_{it} + \nu)(1 - \beta)S_{st}(\alpha, x, 1) \\ & + \nu(1 - \beta) \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1), 0\}) + (1 - \beta)\dot{S}_{st}(\alpha, x, 1) \end{aligned}$$

4 Wages under lockdown

Recovered individuals

During lockdown, the job value for a match that is unlocked with a recovered individual at an arbitrary wage w is equal to:

$$\begin{aligned}(r+\delta+\eta+\nu+\Lambda)J^L_{rt}(w,\alpha,x;U) = p(\alpha,x,1) - w + \nu(1-\beta)\max\{S^L_{rt}(\alpha,x;U),0\}\\ + \Lambda J_{rt}(w,\alpha,x) + \dot{J}^L_{rt}(w,\alpha,x;U)\end{aligned}$$

Re-arranging terms and substituting in the Nash splitting condition, $J^L_{rt}(w,\alpha,x;U) = (1-\beta)S^L_{rt}(\alpha,x;U)$, the wage for a recovered individual in an unlocked job during lockdown takes the following form:

$$\begin{aligned}w_{rt}(\alpha,x;U) = p(\alpha,x,1) + \nu(1-\beta)\max\{S^L_{rt}(\alpha,x;U),0\} - (r+\delta+\eta+\nu+\Lambda)(1-\beta)S^L_{rt}(\alpha,x;U)\\ + \Lambda(1-\beta)S_{rt}(\alpha,x) + (1-\beta)\dot{S}^L_{rt}(\alpha,x;U)\end{aligned}$$

Similarly, the wage equation for a recovered individual in a locked job is equal to:

$$\begin{aligned}w_{rt}(\alpha,x;L) = p(\alpha,x,0) + \nu(1-\beta)\max\{S^L_{rt}(\alpha,x;L),0\} - (r+\delta+\eta+\nu+\Lambda)(1-\beta)S^L_{rt}(\alpha,x;L)\\ + \Lambda(1-\beta)S_{rt}(\alpha,x) + (1-\beta)\dot{S}^L_{rt}(\alpha,x;L)\end{aligned}$$

Infected individuals

The infected never start new jobs (by assumption) and are on sick leave. They inherit their previous wages and their lockdown status. However, they can be hit by re-negotiation shock. Given the job value for a locked match with an infected worker at an arbitrary wage w, we can write:

$$\begin{aligned}w = \rho_y J^L_{rt}(w,\alpha,x;L) + \nu(1-\beta)\max\{S^L_{it}(\alpha,x;L),0\}\\ + \Lambda J_{it}(w,\alpha,x) + \dot{J}^L_{it}(w,\alpha,x;L) - (r+\rho_y+\gamma_y(\ell_{it})+\delta+\eta+\nu+\Lambda)J^L_{it}(w,\alpha,x;L)\end{aligned}$$

Substituting again the Nash splitting condition $J^L_{it}(w,\alpha,x;L) = (1-\beta)S^L_{it}(w,\alpha,x;L)$, we get the following wage equation for an infected worker in a locked match:

$$\begin{aligned}w_{it}(\alpha,x;L) = \rho_y(1-\beta)S^L_{rt}(\alpha,x;L) + \nu(1-\beta)\max\{S^L_{it}(\alpha,x;L),0\} + \Lambda(1-\beta)S_{it}(\alpha,x)\\ - (r+\rho_y+\gamma_y(\ell_{it})+\delta+\eta+\Lambda)(1-\beta)S^L_{it}(\alpha,x;L) + (1-\beta)\dot{S}^L_{it}(\alpha,x;L)\end{aligned}$$

Similarly, the wage equation for an infected worker in an unlocked match during lockdown is equal to:

$$\begin{aligned}w_{it}(\alpha,x;U) = \rho_y(1-\beta)S^L_{rt}(\alpha,x;U) + \nu(1-\beta)\max\{S^L_{it}(\alpha,x;U),0\} + \Lambda(1-\beta)S_{it}(\alpha,x)\\ - (r+\rho_y+\gamma_y(\ell_{it})+\delta+\eta+\Lambda)(1-\beta)S^L_{it}(\alpha,x;U) + (1-\beta)\dot{S}^L_{it}(\alpha,x;U)\end{aligned}$$

Susceptible individuals

An arbitrary wage w to a susceptible individual leaves the employer in a locked match with the following value:

$$\begin{aligned} rJ^L_{st}(w,\alpha,x,m;L) = p(\alpha,x,0) - w &+ (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,m;L)) \\ &+ \lambda^L_{0y}\ell_{it}\left(J^L_{it}(w,\alpha,x;L) - J^L_{st}(w,\alpha,x,m;L)\right) \\ &+ \nu\left((1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L),0\} + V^L_t - J^L_{st}(w,\alpha,x,m;L)\right) \\ &+ \Lambda(J_{st}(w,\alpha,x,m) - J^L_{st}(w,\alpha,x,m;L)) + \dot{J}_{st}(w,\alpha,x,m;L) \end{aligned}$$

for $m \in \{0,1\}$. Imposing free entry $V^L_t = 0$, and re-arranging terms, we get:

$$\begin{aligned} w = p(\alpha,x,0) - (r+\delta+\eta+\lambda^L_{0y}\ell_{it}+\nu+\Lambda)J^L_{st}(w,\alpha,x,m;L) \\ + \lambda^L_{0y}\ell_{it}J^L_{it}(w,\alpha,x;L) \\ + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L),0\} \\ + \Lambda J_{st}(w,\alpha,x,m) + \dot{J}_{st}(w,\alpha,x,m;L) \end{aligned}$$

Substituting the Nash splitting rule $J^L_{st}(w,\alpha,x,m;L) = (1-\beta)S^L_{st}(\alpha,x,m;L)$, the wage for a susceptible worker with a contractual $m \in \{0,1\}$ in a locked match can be written as follows:

$$\begin{aligned} w_{st}(\alpha,x,m;L) = p(\alpha,x,0) &- (r+\delta+\eta+\lambda^L_{0y}\ell_{it}+\nu+\Lambda)(1-\beta)S^L_{st}(\alpha,x,m;L) \\ &+ \lambda^L_{0y}\ell_{it}(1-\beta)S^L_{it}(\alpha,x;L) \\ &+ \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L),0\} \\ &+ \Lambda(1-\beta)S_{st}(\alpha,x,m) + (1-\beta)\dot{S}_{st}(\alpha,x,m;L) \end{aligned}$$

Consider now an unlocked match. An arbitrary wage w to a susceptible individual leaves the employer with the following value:

$$\begin{aligned} (r+\delta+\eta+\lambda^L_{0y}\ell_{it}+\nu+\Lambda)J^L_{st}(w,\alpha,x,0;U) = p(\alpha,x,0) - w + \lambda^L_{0y}\ell_{it}J^L_{it}(w,\alpha,x;U) \\ + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U),0\} \\ + \Lambda J_{st}(w,\alpha,x,0) + \dot{J}_{st}(w,\alpha,x,0;U) \end{aligned}$$

when $m = 0$, and with a value equal to:

$$\begin{aligned} (r+\delta+\eta+(\lambda^L_{0y}+\lambda_{1y})\ell_{it}+\nu+\Lambda)J^L_{st}(w,\alpha,x,1;U) = p(\alpha,x,1) - w \\ + (\lambda^L_{0y}+\lambda_{1y})\ell_{it}J^L_{it}(w,\alpha,x;U) \\ + \nu(1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U),0\} \\ + \Lambda J_{st}(w,\alpha,x,1) + \dot{J}_{st}(w,\alpha,x,1;U) \end{aligned}$$

when $m = 1$. Again, imposing free entry $V_t^L = 0$, substituting the Nash splitting rule $J_{st}^L(w, \alpha, x, m; U) = (1-\beta)S_{st}^L(\alpha, x, m; U)$ and re-arranging terms, we get a wage equation for a susceptible worker in an unlocked match working only from home during lockdown equal to:

$$\begin{aligned} w_{st}(\alpha, x, 0; U) = p(\alpha, x, 0) &+ \lambda_{0y}^L \ell_{it}(1-\beta)S_{it}^L(\alpha, x; U) \\ &- (r + \delta + \eta + \lambda_{0y}^L \ell_{it} + \nu + \Lambda)(1-\beta)S_{st}^L(\alpha, x, 0; U) \\ &+ \nu(1-\beta)\max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} \\ &+ \Lambda(1-\beta)S_{st}(\alpha, x, 0) + (1-\beta)\dot{S}_{st}(\alpha, x, 0; U) \end{aligned}$$

and a wage equation for a susceptible worker in an unlocked match working also away from home during lockdown equal to:

$$\begin{aligned} w_{st}(\alpha, x, 1; U) = p(\alpha, x, 1) &+ (\lambda_{0y}^L + \lambda_{1y})\ell_{it}(1-\beta)S_{it}^L(\alpha, x; U) \\ &- (r + \delta + \eta + (\lambda_{0y}^L + \lambda_{1y})\ell_{it} + \nu + \Lambda)(1-\beta)S_{st}^L(\alpha, x, 1; U) \\ &+ \nu(1-\beta)\max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} \\ &+ \Lambda(1-\beta)S_{st}(\alpha, x, 1) + (1-\beta)\dot{S}_{st}(\alpha, x, 1; U) \end{aligned}$$

References

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