Supplementary Material of
"Trade and Labor Market Institutions: A Tale of Two Liberalizations"

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Appendix A. Details on Aggregate Evidence

A.1 Robustness and further aggregate evidence

In what follows I report a set of robustness checks and further aggregate evidence on the relation between trade regimes, labor market institutions and unemployment. The main results always go through: (1) unemployment is significantly higher after a trade reform, (2) the marginal effect of a trade reform on unemployment is significantly higher the greater the statutory minimum wage, and the lower the costs of worker dismissal in place at the time of openings, (3) cross-country variation in labor market institutions can explain the unemployment response after a trade liberalization.

Different sub-samples. The first robustness is about sample selection. In particular, I split the sample into LAC and non-LAC countries, and estimate equations (1) and (2) in the main text separately for the two sub-sample. Table 1 reports the OLS coefficients. Robust standard errors, clustered at country level, are reported in parenthesis.

De-facto liberalization. The second robustness is about the date of trade reform. Following Wacziarg and Weich (2003), I construct a *de-facto* liberalization date, which combines the five criteria used in Sachs and Warner (1995) together with a sixth indicator, taking value one from the first year a country experience a five percent growth in trade openness (measured by the sum of total exports and imports over GDP), onward. Tables 2 and 3 display the OLS estimates of equations (1) and (2) in the main text using instead this new indicator. Robust standard errors, clustered at country level, are reported in parenthesis. Using *de-facto* liberalization dates does not alter the main results of the paper: (1) the unemployment

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Table 1: **Robustness check 1 - Sub-samples**

	unemp$_{it}$			
VARIABLES	(1.1)	(1.2)	(1.3)	(1.4)
	LAC countries			
Liberalization Dummy				
$\mathbf{1}_{\{t \geq t_i^*\}}$	3.149	2.793	3.324	0.168
	[1.500]*	[1.491]*	[1.403]**	[3.390]
Liberalization Dummy × UI				
$\mathbf{1}_{\{t \geq t_i^*\}}\text{ub}_i$				0.419
				[0.385]
Liberalization Dummy × Minimum wage				
$\mathbf{1}_{\{t \geq t_i^*\}}\underline{\text{w}}_i$				4.078
				[7.045]
Liberalization Dummy × EPL				
$\mathbf{1}_{\{t \geq t_i^*\}}\text{epl}_i$				-0.280
				[0.161]
Observation	466	466	463	453
R-squared	0.154	0.550	0.663	0.755
	non-LAC countries			
Liberalization Dummy				
$\mathbf{1}_{\{t \geq t_i^*\}}$	2.268	2.232	1.543	0.118
	[0.718]**	[0.683]***	[0.563]**	[1.328]
Liberalization Dummy × UI				
$\mathbf{1}_{\{t \geq t_i^*\}}\text{ub}_i$				0.044
				[0.065]
Liberalization Dummy × Minimum wage				
$\mathbf{1}_{\{t \geq t_i^*\}}\underline{\text{w}}_i$				8.159
				[3.555]**
Liberalization Dummy × EPL				
$\mathbf{1}_{\{t \geq t_i^*\}}\text{epl}_i$				-0.335
				[0.179]*
R-squared	0.381	0.562	0.636	0.686
Observations	538	538	535	411
Country FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
Country trend	no	yes	yes	yes
Controls	no	no	yes	yes

Note: unemp$_{it}$ refers to the unemployment rate in country i at time t. $\mathbf{1}_{\{t \geq t_i^*\}}$ is a country-specific dummy variable taking value one in each period after the trade liberalization. Controls include population growth, real GDP per capita and its square, real GDP per capita growth, employment growth, investment share of GDP, the rate of price inflation on household consumption goods, the market exchange rate of the national currency w.r.t the US dollar, and indicators for the occurrence of banking, currency, and sovereign debt crises.. The estimation refers to the full sample. Robust standard errors are clustered at country level (in parenthesis). Source: ILO-stat, WBI, Penn-Table 9.0 and author′s calculations.
**** p<0.01, ** p<0.05, * p<0.1.*

rate increases on average following a trade reform, (2) the response of unemployment is larger the higher the minimum wage and the lower firing costs and (3) cross-country variation in these three institutions can explain the response of unemployment to a fall in trade costs.

Table 2: **Robustness check 2.1 - De-facto liberalization**

VARIABLES	(1.1)	unemp$_{it}$ (1.2)	(1.3)
		Full-sample	
De-facto Liberalization			
$\mathbf{1}_{\{t \geq t_i^*\}}$	2.114	1.871	2.118
	[0.654]***	[0.664]***	[0.670]***
R-squared	0.164	0.518	0.603
Observations	1004	1004	998
		LAC countries	
De-facto Liberalization			
$\mathbf{1}_{\{t \geq t_i^*\}}$	2.467	2.291	3.218
	[1.365]*	[1.339]	[1.089]***
R-squared	0.141	0.554	0.662
Observations	466	466	463
		non-LAC countries	
De-facto Liberalization			
$\mathbf{1}_{\{t \geq t_i^*\}}$	2.385	2.222	1.747
	[0.732]***	[0.739]***	[0.622]***
R-squared	0.386	0.563	0.641
Observations	538	538	535
Country FE	yes	yes	yes
Year FE	yes	yes	yes
Country trend	no	yes	yes
Controls	no	no	yes

Note: unemp$_{it}$ refers to the unemployment rate in country i at time t. $\mathbf{1}_{\{t \geq t_i^*\}}$ is a country-specific dummy variable taking value one in each period after the ***de-facto*** *trade liberalization. Controls include population growth, real GDP per capita and its square, real GDP per capita growth, employment growth, investment share of GDP, the rate of price inflation on household consumption goods, the market exchange rate of the national currency w.r.t the US dollar, and indicators for the occurrence of banking, currency, and sovereign debt crises.. Robust standard errors are clustered at country level (in parenthesis). Source: ILO-stat, WBI, Penn-Table 9.0 and author′s calculations.*
**** p<0.01, ** p<0.05, * p<0.1.*

Dynamic response. Here I complement the main analysis by looking at the dynamic response of unemployment to a trade reform. To do so, I first estimate a dynamic version of equation (1) in the main text, i.e.

$$\text{unemp}_{it} = \sum_{j \in \{-5,0,5,10,15\}} \alpha_j \mathbf{1}_{\{t \in (t_i^*+j, t_i^*+j+5]\}} + \gamma_t + \upsilon_i + \eta_i (t - t_i^*) + \delta X_{it} + \epsilon_{it} \qquad (1)$$

where t_i^* denotes the liberalization date of country i, whereas $\mathbf{1}_{\{t \in (t_i^*+j, t_i^*+j+5]\}}$ is a dummy

Table 3: **Robustness check 2.2 - De-facto liberalization**

	unemp_{it}			
VARIABLES	(2.1)	(2.2)	(2.3)	(2.4)
De-facto Liberalization				
$\mathbf{1}_{\{t \geq t_i^*\}}$	1.001	-0.101	3.237	0.230
	[0.726]	[1.359]	[1.072]***	[1.812]
De-facto Liberalization × UI				
$\mathbf{1}_{\{t \geq t_i^*\}}\text{ub}_i$	0.219			0.154
	[0.043]***			[0.057]**
De-facto Liberalization × Minimum wage				
$\mathbf{1}_{\{t \geq t_i^*\}}\underline{\text{w}}_i$		5.975		5.293
		[3.302]*		[2.820]*
De-facto Liberalization × EPL				
$\mathbf{1}_{\{t \geq t_i^*\}}\text{epl}_i$			-0.267	-0.223
			[0.111]**	[0.125]*
R-squared	0.632	0.648	0.625	0.687
Observations	841	828	959	734
Country FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
Country trend	yes	yes	yes	yes
Controls	yes	yes	yes	yes
F-test p-value	0.997	0.950	0.985	0.995

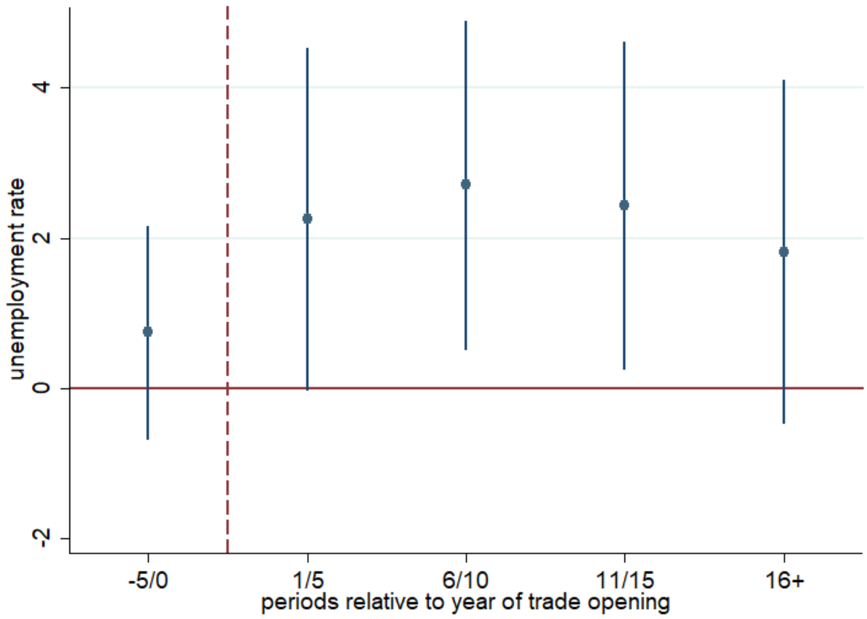
Note: $unemp_{it}$ refers to the unemployment rate in country i at time t. $\mathbf{1}_{\{t \geq t_i^*\}}$ is a country-specific dummy variable taking value one in each period after the ***de-facto*** *trade liberalization, t_i^*. epl_i, ub_i and $\underline{w}_i$ refers to employment legislation, unemployment benefits and minimum wage regulation in place at the time of liberalization. Controls include population growth, real GDP per capita and its square, real GDP per capita growth, employment growth, investment share of GDP, the rate of price inflation on household consumption goods, the market exchange rate of the national currency w.r.t the US dollar, and indicators for the occurrence of banking, currency, and sovereign debt crises.. The p-value refers to the F-test for joint significance of labor market institutions, i.e. $H_0 : \mathbf{1}_{\{t \geq t_i^*\}} + \mathbf{1}_{\{t \geq t_i^*\}} ub_i + \mathbf{1}_{\{t \geq t_i^*\}} \underline{w}_i + \mathbf{1}_{\{t \geq t_i^*\}} epl_i > 0$. Robust standard errors are clustered at country level (in parenthesis). Source: ILO-stat, WBI, Penn-Table 9.0 and author's calculations.*
**** p<0.01, ** p<0.05, * p<0.1.*

variable that takes value one if for any period t between $t_i^* + j$ and $t_i^* + j + 5$. I fit five of these dummies into equation (1), covering any periods $t \in (t_i^* - 5, t_i^* + 20]$, and I set the time spanning between $t_i^* - 20$ and $t_i^* - 5$ as a baseline group. Second, I estimate a dynamic version of equation (2) in the main text, i.e.

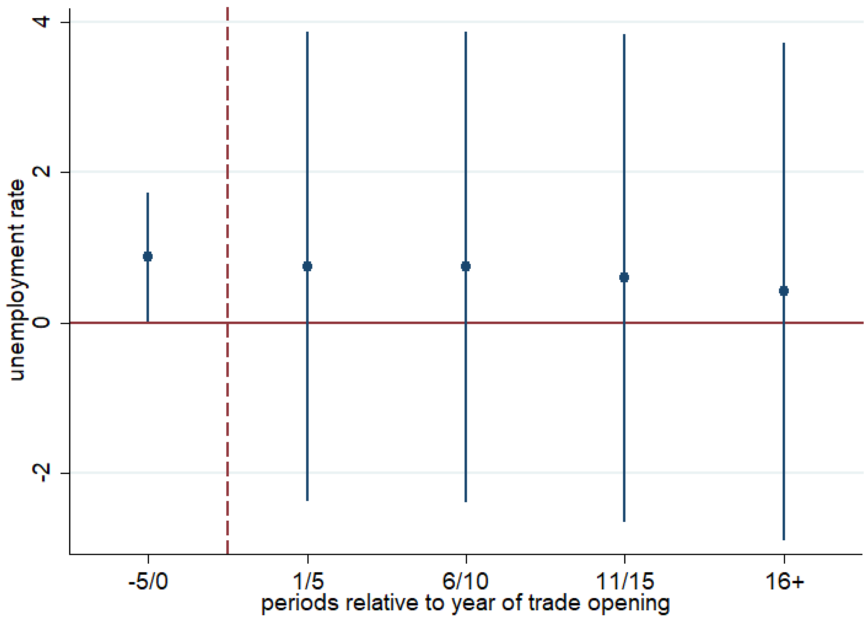
$$\text{unemp}_{it} = \sum_{j \in \{-5,0,5,10,15\}} \alpha_j \mathbf{1}_{\{t \in (t_i^*+j, t_i^*+j+5]\}} + \beta \mathbf{1}_{\{t \geq t_i^*\}} \text{z}_i + \gamma_t + \upsilon_i + \eta_i (t - t_i^*) + \delta X_{it} + \epsilon_{it} \quad (2)$$

Figure 1: **Lead and lagged effect of trade reform**

(a) Baseline



(b) Controlling for labor market institutions



Note: This figure report the OLS estimates of lead and lagged effects of a trade reform on the unemployment rate for the baseline case (equation 1) and after controlling for labor market institutions (equation 2).

where the interaction terms $\mathbf{1}_{\{t \geq t_i^*\}} \mathrm{z}_i$ are now included to capture cross-country differences in unemployment rate in periods of post-liberalization systematically associated to labor

Table 4: **Robustness check 3 - Dynamic response**

	unemp$_{it}$	
VARIABLES	(1.1)	(1.2)
Liberalization Dummy 1		
$\mathbf{1}_{\{t\in(t_i^*-5,t_i^*+j]\}}$	0.721	0.951
	[0.696]	[0.429]***
Liberalization Dummy 2		
$\mathbf{1}_{\{t\in(t_i^*,t_i^*+j+5]\}}$	2.253	0.952
	[1.111]***	[1.494]
Liberalization Dummy 3		
$\mathbf{1}_{\{t\in(t_i^*+5,t_i^*+j+10]\}}$	2.715	0.978
	[1.069]***	[1.502]
Liberalization Dummy 4		
$\mathbf{1}_{\{t\in(t_i^*+10,t_i^*+j+15]\}}$	2.411	0.780
	[1.069]***	[1.580]
Liberalization Dummy 5		
$\mathbf{1}_{\{t>t_i^*+15\}}$	1.803	0.625
	[1.124]	[1.594]
Liberalization Dummy × UI		
$\mathbf{1}_{\{t\geq t_i^*\}}\text{ub}_i$		0.116
		[0.049]***
Liberalization Dummy × Minimum wage		
$\mathbf{1}_{\{t\geq t_i^*\}}\underline{\text{w}}_i$		6.415
		[2.630]***
Liberalization Dummy × EPL		
$\mathbf{1}_{\{t\geq t_i^*\}}\text{epl}_i$		-0.292
		[0.103]***
R-squared	0.607	0.632
Observations	998	784
Country FE	yes	yes
Year FE	yes	yes
Country trend	yes	yes
Controls	yes	yes

Note: unemp$_{it}$ refers to the unemployment rate in country i at time t. $\mathbf{1}_{\{t\geq t_i^*\}}$ is a country-specific dummy variable taking value one in each period after trade liberalization, t_i^*. epl_i, ub_i and $\underline{w}_i$ refers to employment legislation, unemployment benefits and minimum wage regulation in place at the time of liberalization. Controls include population growth, real GDP per capita and its square, real GDP per capita growth, employment growth, investment share of GDP, the rate of price inflation on household consumption goods, the market exchange rate of the national currency w.r.t the US dollar, and indicators for the occurrence of banking, currency, and sovereign debt crises. Robust standard errors are clustered at country level (in parenthesis). Source: ILO-stat, WBI, Penn-Table 9.0 and author's calculations.

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market institutions, z_i.

Figure 1 displays the time-varying marginal effects of a trade reform estimated in equation (1) and (2). In the baseline case (left panel), the coefficients on the trade reform leads are not statistically different than zero, showing little anticipatory response within each country close to embark in trade reform. From the year of adoption onward, the unemployment rate increases by more than two percentage points, reaching its peak between six and ten years after the reform, and declining afterwards. Once controlling for the cross-country variation in minimum wage, EPL and UI at the time of the trade reform, no significant effect on unemployment rate is detected anymore (right panel). Table 4 reports the OLS estimates for the interaction terms between the liberalization dates and the labor market institutions in equation (2).

Appendix B. Details on the Model

B.1. Demand Functions and Firm Revenues

In this section I characterize the demand functions for each variety ω, the revenue functions of domestic firms and the revenue premium of domestic exporting firms. Given the CES structure, the inverse demand for a domestic variety ω at time t from a worker i with income I_i and from a firm j with gross revenues G_j is equal, respectively, to

$$q_{i,t}(\omega) = \gamma \left(\frac{p_t(\omega)}{P_t}\right)^{-\sigma} \frac{I_{i,t}}{P_t}$$

and

$$q_{j,t}(\omega) = (1-\alpha)\frac{\sigma-1}{\sigma} \left(\frac{p_t(\omega)}{P_t}\right)^{-\sigma} \frac{G_{j,t}}{P_t}$$

Combining demands and aggregating across workers and firms, we get,

$$q_t(\omega) = \int_0^1 q_{i,t}(\omega)di + \int_0^{N_{h,t}} q_{j,t}(\omega)dj \implies q_t(\omega) = D_{h,t}p_t(\omega)^{-\sigma}$$

where

$$D_{h,t} = P_t^{\sigma-1}\left[\gamma \int_0^1 I_{i,t}di + (1-\alpha)\frac{\sigma-1}{\sigma}\int_0^{N_{h,t}} G_{j,t}dj\right]$$

denotes the aggregate domestic expenditures. Similarly, the domestic inverse demand for a foreign variety ω^*, read

$$q_{i,t}(\omega^*) = \gamma \left(\frac{\tau_{a,t}\tau_{c,t}k_t p_t(\omega^*)}{P_t} \right)^{-\sigma} \frac{I_{i,t}}{P_t}$$

and

$$q_{j,t}(\omega^*) = (1-\alpha)\frac{\sigma-1}{\sigma} \left(\frac{\tau_{a,t}\tau_{c,t}k_t p_t(\omega^*)}{P_t} \right)^{-\sigma} \frac{G_{j,t}}{P_t}$$

which implies

$$q_t(\omega^*) = \int_0^1 q_{i,t}(\omega^*)di + \int_0^{N_{h,t}} q_{j,t}(\omega^*)dj \implies q_t(\omega^*) = [\tau_{a,t}\tau_{c,t}k_t p_t(\omega^*)]^{-\sigma} D_{h,t} \tag{3}$$

Finally, the foreign demand for domestically produced good ω is given by

$$q_t^*(\omega) = D_{f,t} p_t^*(\omega)^{-\sigma} \tag{4}$$

where $p_t^*(\omega)$ is the price of domestic variety ω in the foreign market while $D_{f,t}$ denotes the aggregate expenditures abroad denominated in foreign currency, net of any effects of foreign commercial policies, and treated as exogenous parameter.

Consider the problem of a domestic firm that produces q_t units of output which are sold in the home market. The gross revenues of this firm at time t are equal to

$$G_{h,t}(\omega) = p_t(\omega)q_t(\omega) \tag{5}$$

From the inverse demand in equation (3) we can solve for the price of variety ω charged in the home market, $p_t(\omega)$, i.e.

$$p_t(\omega) = \left(\frac{q_t(\omega)}{D_{h,t}} \right)^{-\frac{1}{\sigma}}$$

Substituting $p_t(n)$ into equation (5), we obtain the revenues of non-exporting domestic firms,

$$G_{h,t}(\omega) = D_{h,t}^{\frac{1}{\sigma}} q_t(\omega)^{\frac{\sigma-1}{\sigma}} \tag{6}$$

Consider now the problem of a firm located in the home country that produces q_t units of output which are then shipped to the foreign market in addition to the home market. The choice variables for the firm are the fraction is the fraction χ_t of total output allocated to either markets. Because of iceberg trade costs, $\tau_{c,t}$, only $\frac{1}{\tau_{c,t}}\chi_t q_t$ units are exported to the foreign market, where the quantity $\frac{\tau_{c,t}-1}{\tau_{c,t}}\chi_t q_t$ is lost in shipping the good abroad. The gross revenues of this firm at time t are equal to

$$G_{f,t}(\omega) = \max_{\chi_t} \quad p_t(\omega)(1-\chi_t)q_t(\omega) + k_t p_t^*(\omega)\frac{\chi_t q_t(\omega)}{\tau_{c,t}} \tag{7}$$

From the inverse demand in equation (4) we can solve for the price of variety ω charged in the foreign market, $p_t^*(\omega)$, i.e.

$$p_t^*(\omega) = \left(\frac{q_t(\omega)}{D_{f,t}}\right)^{-\frac{1}{\sigma}}$$

Substituting $p_t(\omega)$ and $p_t^*(\omega)$ into equation (7), we obtain the revenues of exporting domestic firms,

$$G_{f,t}(\omega) = \max_{\chi_t} \quad q_t(\omega)^{\frac{\sigma-1}{\sigma}}\left[(1-\chi_t)^{\frac{\sigma-1}{\sigma}} D_{h,t}^{\frac{1}{\sigma}} + k_t\left(\frac{\chi_t}{\tau_{c,t}}\right)^{\frac{\sigma-1}{\sigma}} D_{f,t}^{\frac{1}{\sigma}}\right] \tag{8}$$

The optimal output share allocated to the foreign market is the maximizer of the problem in equation (8), and it reads as

$$\chi_t = \frac{k_t^\sigma D_{f,t}\tau_{c,t}^{1-\sigma}}{D_{h,t} + k_t^\sigma D_{f,t}\tau_{c,t}^{1-\sigma}} \tag{9}$$

Substituting χ_t into equation (8), we can write the revenue function as follows,

$$G_{f,t}(\omega) = q_t(\omega)^{\frac{\sigma-1}{\sigma}}\left[D_{h,t} + k^\sigma \tau_{c,t}^{-(\sigma-1)} D_{f,t}\right]^{\frac{1}{\sigma}} = q_t(\omega)^{\frac{\sigma-1}{\sigma}} D_{h,t}^{\frac{1}{\sigma}}[1+d_{f,t}]^{\frac{1}{\sigma}} \tag{10}$$

where $d_{f,t}$ is the revenue premium from exporting, equal to

$$d_{f,t} = k_t^\sigma \tau_{c,t}^{-(\sigma-1)}\frac{D_{f,t}}{D_{h,t}} \tag{11}$$

Finally, combining equations (9) and (8), I can write the optimal share of output allocated to the foreign market as

$$\chi_t = 1 - [1+d_{f,t}]^{-\sigma} \tag{12}$$

B.2. Intermediate expenditure

Intermediate inputs are chosen every period so to maximize the net revenue function.

This implies the following optimization problem for a generic firm,

$$R_t(z,\ell) = \max_{m} \quad G_t(q(z,m,\ell)) - P_t m \tag{13}$$

where $G_t(q(z,m,\ell))$ denotes the gross revenue function,

$$G_t(q(z,m,\ell)) = D_{h,t}^{\frac{1}{\sigma}}[1+\mathbf{1}_t^x d_{f,t}]^{\frac{1}{\sigma}} q(z,m,\ell)^{\frac{\sigma-1}{\sigma}} - c_x \mathbf{1}_t^x \tag{14}$$

whereas $q(z,m,\ell)$ is the production function,

$$q(z,m,\ell) = z m^{1-\alpha}\ell^{\alpha} \tag{15}$$

Material expenditure satisfies the following first order condition,

$$P_t m_t = (1-\alpha)\frac{(\sigma-1)}{\sigma} G_t(q(z,m,\ell))$$

Solving for m_t and substituting into equation (13), yields the following expression for the net revenue function,

$$R_t(z,\ell) = \Delta_t(z,\ell)(z\ell^{\alpha})^{\frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)}} \tag{16}$$

where $\Delta_t(z,\ell)$ is equal to

$$\Delta_t(z,\ell) = \frac{1-(1-\alpha)\frac{(\sigma-1)}{\sigma}}{\left((1-\alpha)\frac{(\sigma-1)}{\sigma}\right)^{1-\frac{\sigma}{\sigma-(1-\alpha)(\sigma-1)}}} \left[\frac{D_{h,t} + \mathbf{1}_t^x(z,\ell)k^{\sigma}\tau_{c,t}^{-(\sigma-1)} D_{f,t}}{P_t^{-(1-\alpha)(\sigma-1)}}\right]^{\frac{1}{\sigma-(1-\alpha)(\sigma-1)}} \tag{17}$$

B.3. Firms optimal policies

Employment Policy. Standard optimization arguments lead to the two following first order conditions for hires and separation of an active firm at time t:

$$\frac{\partial \pi_t(z',\ell,\ell')}{\partial \ell'} + \frac{\partial V_{t+1}(z',\ell')}{\partial \ell'} = 0$$

$$\implies \frac{\partial R_t(z',\ell')}{\partial \ell'} + \frac{\partial V_{t+1}(z',\ell')}{\partial \ell'} = \frac{\partial w_t(z',\ell')\ell'}{\partial \ell'} + \mathbf{1}_t^h(z',\ell)\frac{\partial C_t^h(\ell,\ell')}{\partial \ell'} - \mathbf{1}_t^f(z',\ell)c_{f,t}$$

where $\mathbf{1}_t^h(z',\ell)$ and $\mathbf{1}_t^f(z',\ell)$ are two indicator functions taking value one if the firm is, respectively, hiring and firing and zero otherwise, $\frac{\partial R_t(z',\ell')}{\partial \ell'}$ denotes marginal revenues while $\frac{\partial V_{t+1}(z',\ell')}{\partial \ell'}$ captures the marginal effect of current employment decisions on the continuation value of the firm. This equation has a straightforward interpretation: firm will expand or contract up to the point where current and future marginal benefits from resizing is equal to the relative marginal costs, captured by the marginal effect on wage payments, $\left(\frac{\partial w_t(z',\ell')\ell'}{\partial \ell'}\right)$ plus the marginal costs of hiring new workers, $\left(\frac{\partial C_t^h(\ell,\ell')}{\partial \ell'}\right)$ or firing some of them $(c_{f,t})$. As in Cooper et al. (2007) and Elsby and Michaels (2013), the presence of adjustment costs make the optimal employment decisions of the firm be characterized by two reservation thresholds, $z^H(\ell)$ and $z^F(\ell)$, which are defined by the following two equations:

$$\frac{\partial R_t(z^H(\ell),\ell')}{\partial \ell'} - \frac{\partial w_t(z^H(\ell),\ell')\ell'}{\partial \ell'} + \frac{\partial V_{t+1}(z^H(\ell),\ell')}{\partial \ell'} = \frac{\partial C_t^h(\ell,\ell')}{\partial \ell'}$$
$$\frac{\partial R_t(z^F(\ell),\ell')}{\partial \ell'} - \frac{\partial w_t(z^F(\ell),\ell')\ell'}{\partial \ell'} + \frac{\partial V_{t+1}(z^F(\ell),\ell')}{\partial \ell'} = -c_{f,t}$$

The derivative of the continuation value of the marginal worker can be written as

$$\frac{\partial V_{t+1}(z',\ell')}{\partial \ell'} = \frac{1-\delta}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') \left(\frac{\partial \mathbf{E}_{z''|z'} \max_{\ell''} [\pi_{t+1}(z'',\ell',\ell'') + V_{t+1}(z'',\ell'')]}{\partial \ell'} \right)$$

which, by the envelope theorem, reads as

$$\begin{aligned}\frac{\partial V_{t+1}(z',\ell')}{\partial \ell'} &= \frac{1-\delta}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') \mathbf{E}_{z''|z'} \mathbf{1}_{t+1}^h(z'',\ell') \frac{\partial C_{t+1}^h(\ell',\ell'')}{\partial \ell'} \\ &\quad - \frac{1-\delta}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') \mathbf{E}_{z''|z'} \mathbf{1}_{t+1}^f(z'',\ell') c_{f,t+1} \\ &\quad + \frac{1-\delta}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') \mathbf{E}_{z''|z'} (1-\mathbf{1}_{t+1}^h(z'',\ell'))(1-\mathbf{1}_{t+1}^f(z'',\ell')) \left[\frac{\partial \pi_{t+1}(z'',\ell',\ell')}{\partial \ell'} + \frac{\partial V_{t+2}(z'',\ell')}{\partial \ell'} \right]\end{aligned}$$

or equivalently

$$\frac{\partial V_{t+1}(z',\ell')}{\partial \ell'} = \begin{cases} -\frac{(1-\delta)}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') c_{f,t+1}, & \text{if} \quad z' < z^F(\ell) \\ \frac{(1-\delta)}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') \mathbf{E}_{z''|z} \left[\frac{\partial \pi_{t+1}(z'',\ell',\ell')}{\partial \ell'} + \frac{\partial V_{t+2}(z'',\ell')}{\partial \ell'} \right], & \text{if} \quad z' \in [z^H(\ell), z^H(\ell)] \\ \frac{(1-\delta)}{1+r_{t+1}} \mathbf{1}_{t+1}^o(z',\ell') \frac{\partial C_{t+1}^h(\ell',\ell'')}{\partial \ell'}, & \text{if} \quad z' > z^H(\ell) \end{cases}$$

Consider an incumbent firm entering the period with l employees and receiving an idiosyncratic productivity shock z'. The optimal employment level in the current period, $L_t(z', \ell)$, is thus characterized by the following policy function,

$$L_t(z', \ell) = \begin{cases} \ell^F(z') = \ell(z^F(\ell), \ell), & \text{if} \quad z' < z^F(\ell) \\ \ell, & \text{if} \quad z' \in [z^F(\ell), z^H(\ell)] \\ \ell^H(z') = \ell(z^H(\ell), \ell), & \text{if} \quad z' > z^H(\ell) \end{cases}$$

where $\ell^F(z')$ and $\ell^H(z')$ refer to the optimal employment level consistent with the optimality conditions,

$$\frac{\partial R_t(z', \ell')}{\partial \ell'}|_{\ell'=\ell^H} - \frac{\partial w_t(z', \ell')\ell'}{\partial \ell'}|_{\ell'=\ell^H} + \frac{\partial V_{t+1}(z', \ell')}{\partial \ell'}|_{\ell'=\ell^H} = \frac{\partial C_t^h(\ell, \ell')}{\partial \ell'}|_{\ell'=\ell^H} \tag{18}$$

$$\frac{\partial R_t(z', \ell')}{\partial \ell'}|_{\ell'=\ell^F} - \frac{\partial w_t(z', \ell')\ell'}{\partial \ell'}|_{\ell'=\ell^F} + \frac{\partial V_{t+1}(z', \ell')}{\partial \ell'}|_{\ell'=\ell^F} = -c_{f,t} \tag{19}$$

Therefore, if the idiosyncratic productivity z' is below the reservation threshold, $z^F(\ell)$, the firm will fire workers, pushing up the marginal benefits from employment till it is equal to marginal cost of dismissal. The opposite will happen if z' is above the reservation threshold, $z^H(\ell)$: the firm will hire workers, driving down the marginal return from employment till it is equal to the marginal cost of hiring. On the other hand, if z' lied between the two thresholds, the firm will be inactive, and will set current employment level equal to the previous one.

Export Policy. Each period t, incumbent firms decide whether to sell their product abroad or not. Export is a static decision, it entails the payment of a fixed costs, c_x, and grant a revenue premium, $d_{f,t}$. The presence of a fixed cost of exporting makes the optimal export participation decision, $\mathbf{1}_t^x(z', \ell')$ be characterized by a threshold productivity level, $z^x(l')$, which is implicitly defined as a solution the following equation,

$$R_{h,t}(z^x(\ell'), \ell') = R_{x,t}(z^x(\ell'), \ell') - c_x$$

so that

$$\mathbf{1}_t^x(z', \ell') = \begin{cases} 1, & \text{if} \quad z' \geq z^x(\ell') \\ 0, & \text{otherwise} \end{cases}$$

Exit Policy. At the beginning of each period t, firms who did not exit the industry for exogenous reasons decide whether to continue to operate or not. The presence of a fixed cost of operation, c^o, together with the autocorrelation of the idiosyncratic productivity process, makes the optimal exit decision be characterized by a threshold productivity level, $z^O(\ell)$, which is defined by the following equation

$$\int_{z' \in \mathcal{Z}} \tilde{V}_t(z', \ell)\Gamma(z'|z^O(\ell)) = c^o$$

Consider a firm entering the period with l employees and receiving an idiosyncratic productivity shock z'. The optimal exit strategy, $\mathbf{1}^o_t(z, \ell)$, is thus characterized by the following policy function:

$$\mathbf{1}^o_t(z, \ell) = \begin{cases} 1, & \text{if} \quad z \geq z^O(\ell) \\ 0, & \text{otherwise} \end{cases}$$

B.4. Wage Determination

Wages of industrial employees are determined using Binmore et al. (1986) bargaining solution, generalized to a setting when marginal returns are diminishing. Firms and workers meet and bargain simultaneously and on a one-to-one basis. Each worker is treated marginally by the firm. Failing to reach an agreement would imply a loss for the firm (who cannot recover back the costs of posting vacancies and cannot contact other workers in the current period to replace the existing ones) and for workers (who would forgo current wage payments). The threats of a temporary disruption of production due to a breakdown of negotiations generates a surplus to split between parties, which is equal to the marginal flow surplus. Solution for the wage paid to employees by a hiring firm is implicitly defined by the following sharing rule,

$$\beta\Pi^{\text{firm}}_t(z', \ell') = (1 - \beta)\Pi^{\text{worker}}_t(z', \ell') \tag{20}$$

where $\Pi^{\text{firm}}_t(z', \ell')$ is the firm marginal flow surplus, defined as,

$$\Pi^{\text{firm}}_t(z', \ell') = \frac{\partial R_t(z', \ell')}{\partial \ell'} - \frac{\partial w_t(z', \ell')\ell'}{\partial \ell'} \tag{21}$$

while $\Pi^{\text{worker}}_t(z', \ell',)$ is the worker marginal flow surplus equal to

$$\Pi^{\text{worker}}_t(z', \ell') = w_t(z', \ell') - b - b^u_t \tag{22}$$

Substituting the surplus functions into the sharing rule, one obtains a first-order partial differential equation,

$$w_t(z',\ell') = (1-\beta)(b+b_t^u) + \beta\left(\frac{\partial R_t(z',\ell')}{\partial \ell'} - \frac{\partial w_t(z',\ell')}{\partial \ell'}\ell'\right) \tag{23}$$

Re-arranging the differential equation, we get

$$\frac{\partial w_t(z',\ell')}{\partial \ell'} + \frac{w_t(z',\ell')}{\beta\ell'} - \left[\frac{\partial R_t(z',\ell')}{\partial \ell'} - (1-\beta)(b+b_t^u)\right]\frac{1}{\ell'} = 0 \tag{24}$$

Suppressing for easy of notation the dependence from z', equation (24) can be re-written in the following form:

$$\frac{\partial y(\ell')}{\partial \ell'} + p(\ell')y(\ell') + q(\ell') = 0 \tag{25}$$

where

$$\begin{aligned} y(\ell') &= w_t(z',\ell') \\ p(\ell') &= \frac{1}{\beta\ell'} \\ q(\ell') &= -\left[\frac{\partial R_t(z',\ell')}{\partial \ell'} - (1-\beta)(b+b_t^u)\right]\frac{1}{\ell'} \end{aligned} \tag{26}$$

Plugging the expressions in (27) into the solution of (25), one can express wages as

$$w_t(z',\ell') = (\ell')^{-\frac{1}{\beta}} \int_0^{\ell'} x^{\frac{1-\beta}{\beta}} \frac{\partial R_t(z',x)}{\partial x} dx \tag{27}$$

Substituting the definition of marginal revenue function into (25) and integrating over employment yields the wage expression in the text

$$w_t(z',\ell') = (1-\beta)(b+b_t^u) + \frac{\beta}{1-\beta+\alpha\beta\Lambda}\frac{\partial R_t(z',\ell')}{\partial \ell} \tag{28}$$

with $\Lambda = \frac{\alpha(\sigma-1)}{\sigma-(1-\alpha)(\sigma-1)} > 0$. Notice that equation (28) generalizes the solution obtained in Cosar et al. (2016).